16TH EAST ASIAN ACTUARIAL CONFERENCE
VENTURE INTO UNCERTAINTY
CAPTURE OPPORTUNITIES
10-13 OCTOBER 2011 ▪ KUALA LUMPUR, MALAYSIA

Pricing Guaranteed Annuity Option with Surrender Rates Consideration

Shing-Her Juang
Department of Financial Engineering & Actuarial Mathematics, Soochow University, Taiwan

Shu-Chuan Chen
Actuarial Department, Bank Taiwan Life Insurance

Chih-Hua Chiao
Department of Financial Engineering & Actuarial Mathematics, Soochow University, Taiwan

Po-Kai Shen
Product Development & Management Department,
Nan Shan Life Insurance

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Agenda

- Motivation
- Introduction
- Models
- Numerical Illustrations
- Summary

Motivation

- Aging society
- Low birth rate, improving medical technology, longer life expectancy
- Taiwan becomes an aged society in 2017
- Economical safety is a major concern for older generation
- Economic burden for younger generation

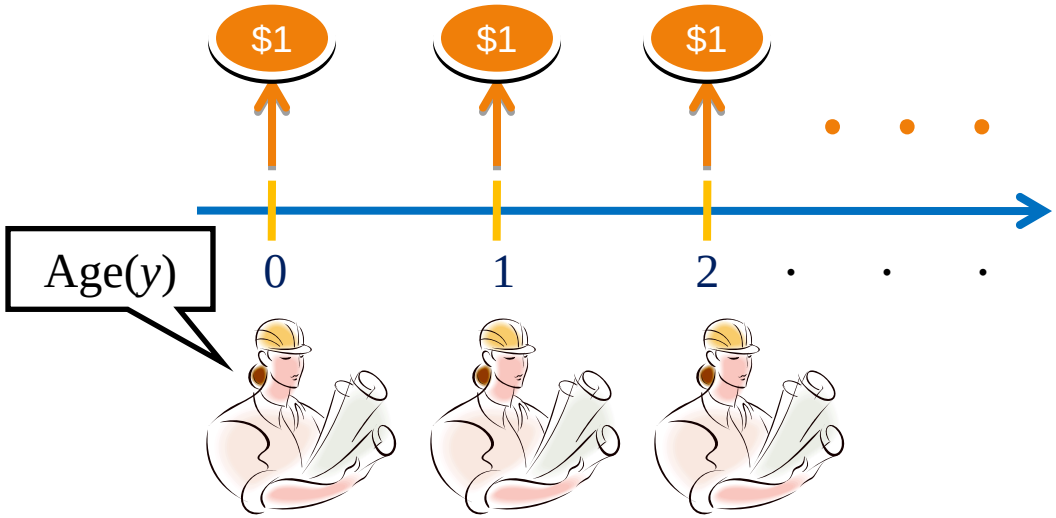
Motivation (cont.)

- Life annuity
- Guaranteed annuity option
- Dynamic surrender model
- Fair pricing

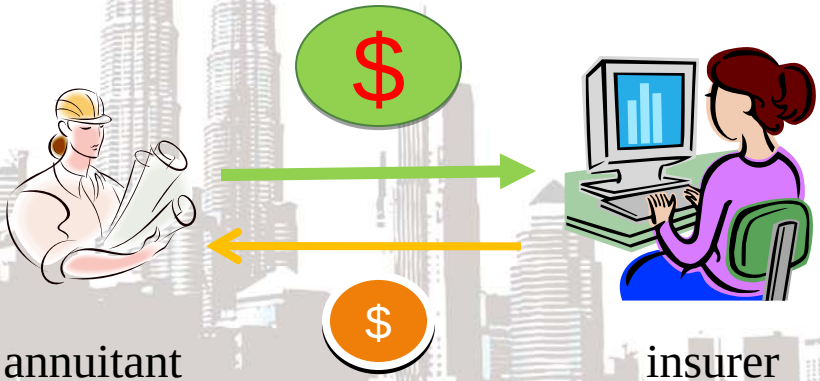
Life annuity due

- Life annuity due

$$\ddot{a}_y = \sum_{t=0}^{\omega-y} \$1 \times {}_t p_y \times (1+i)^{-t}$$



- Annuity purchase rate (APR)

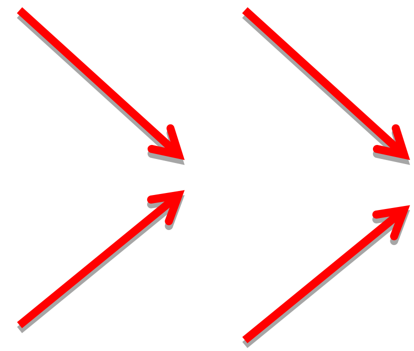


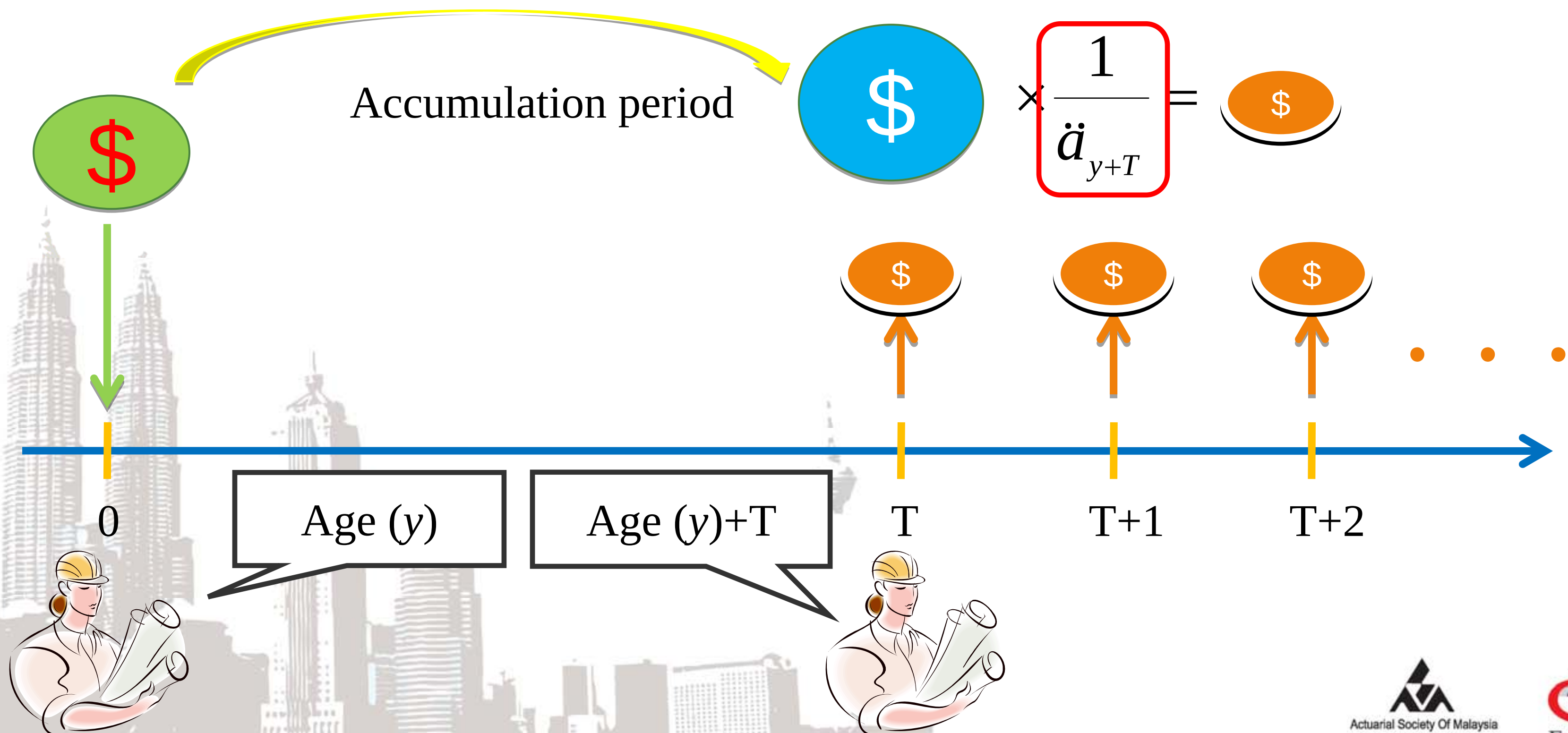
$$\$ = \$\ \ddot{a}_y$$

$$\Rightarrow \$ \frac{1}{\ddot{a}_y} = \$$$

Single premium - deferred life annuity

- Risk factors
  - Life table
  - Interest rate





Guarantee annuity option

- Guarantee annuity option (GAO)

APR

1000

$$\times \frac{1}{\ddot{a}_{y+T}}\left(\frac{1}{10}\right)=100$$

$$\times GAR\left(\frac{1}{9}\right)=111$$

$ × $\frac{1}{\ddot{a}_{y+T}}$ = $

$ × GAR = $

$

Accumulation period

Guarantee Annuity Rate (GAR)

$ $ $ $

0 T T+1 T+2 . . .

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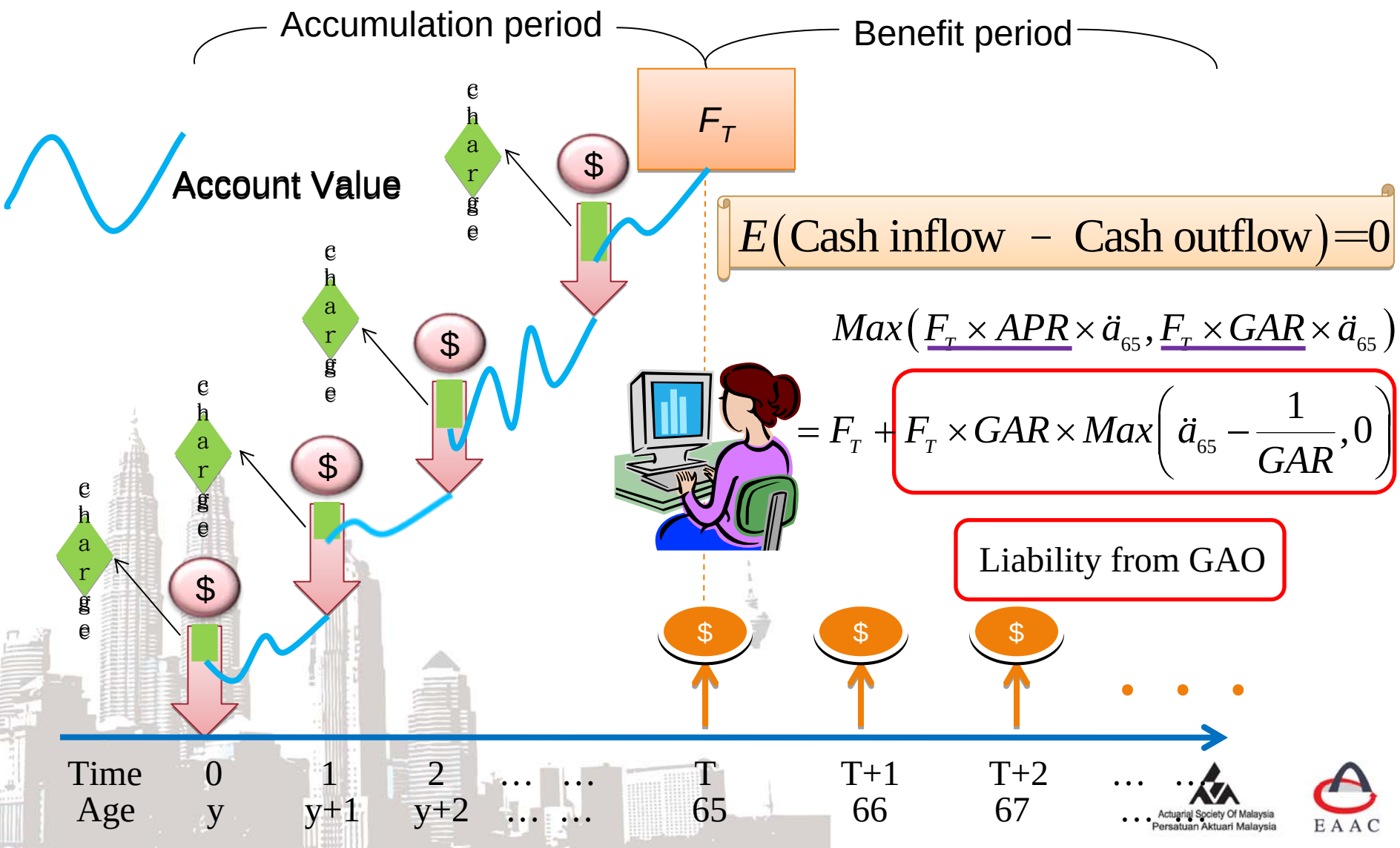
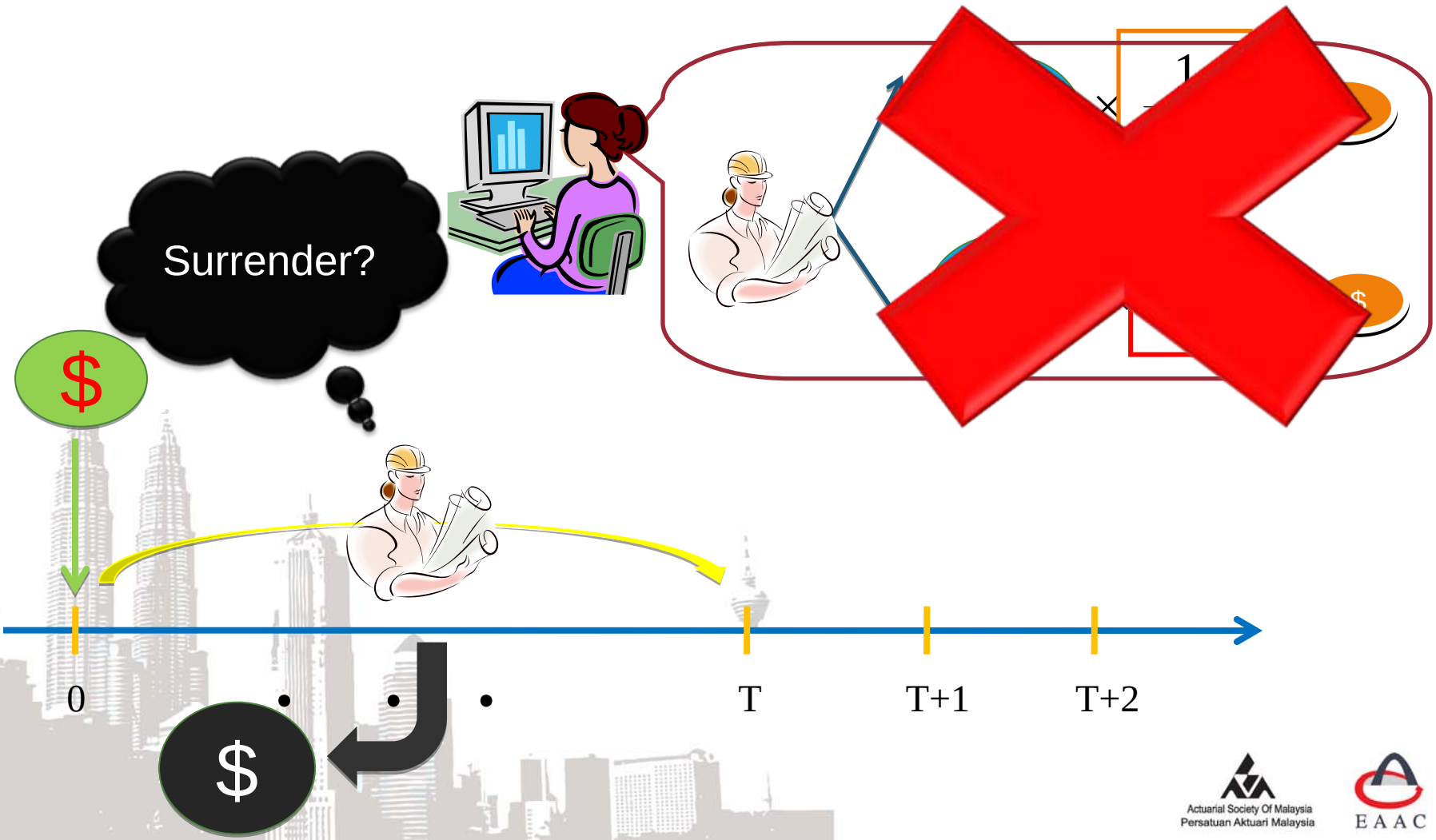
Level premium



Surrender



Model Building

- Yang and Tang (2006) - Risk neutral measure

S_t: Asset price at time t

r_t: interest rate at time t

$$\ddot{a}_{65} = \sum_{t=0}^{\omega-65} \$1 \times {}_t p_{65} \times P(65, 65+t)$$

$$\frac{dS_t}{S_t} = r_t dt + \sigma_S dW_t^{Q,S}$$

$$dr_t = \hat{\kappa}\left(\hat{\theta} - r_t\right)dt + \sigma_r \sqrt{r_t}\, dW_t^{Q,r}$$

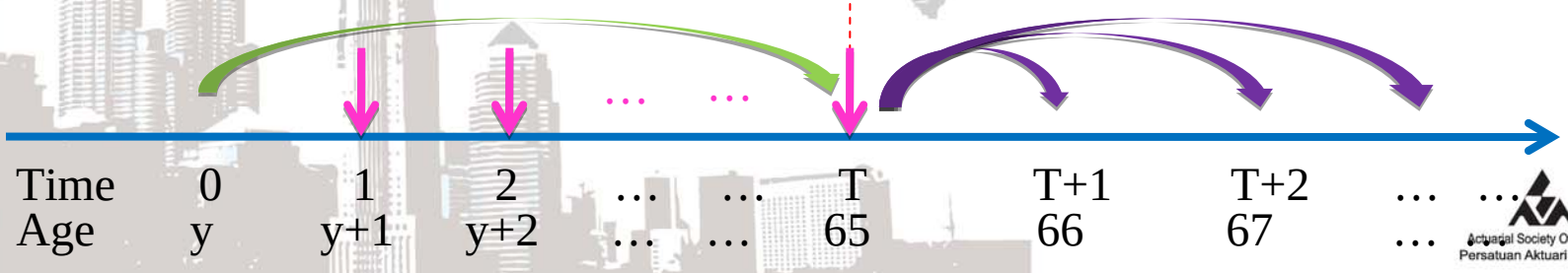
$$dW_t^{Q,S} dW_t^{Q,r} = \rho dt$$

$$P(65, 65+t) = A(65, 65+t)\exp\left[-B(65, 65+t) \times r_{65}\right]$$

$$A(65, 65+t) = \left[\frac{2\gamma e^{(\gamma+\hat{\kappa})\frac{t}{2}}}{(\gamma+\hat{\kappa})\left(e^{\gamma \times t} - 1\right) + 2\gamma}\right]^{\frac{2\hat{\kappa}\hat{\theta}}{\sigma_r^2}}$$

$$\mathrm{B}(65, 65+t) = \frac{2\left(e^{\gamma \times t} - 1\right)}{(\gamma+\hat{\kappa})\left(e^{\gamma \times t} - 1\right) + 2\gamma}$$

$$\gamma = \sqrt{\hat{\kappa}^2 + 2\sigma_r^2}$$



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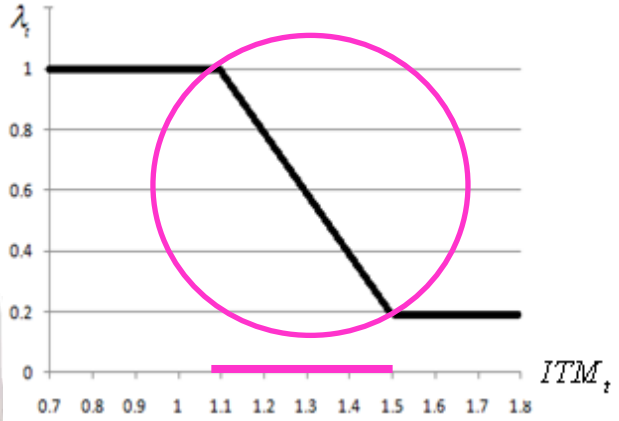
Surrender model

- AAA

$$\lambda_t = Min\left\{1, Max\left[0.5, 1-1.25\left(ITM_t - 1.1\right)\right]\right\}$$

$$\lambda_t : \text{adjust coefficient},\ ITM_t = \frac{\text{Guaranteed value at time } t}{\text{Account value at time } t}$$

- λ_t and ITM_t



- Surrender rate

$${}_{t|}q_x^{(l)}\left(\text{Surrender rate at time } t\right) = {}_{t|}q_x^{(l)base}\left(\text{Experience rate}\right) \times \lambda_t$$

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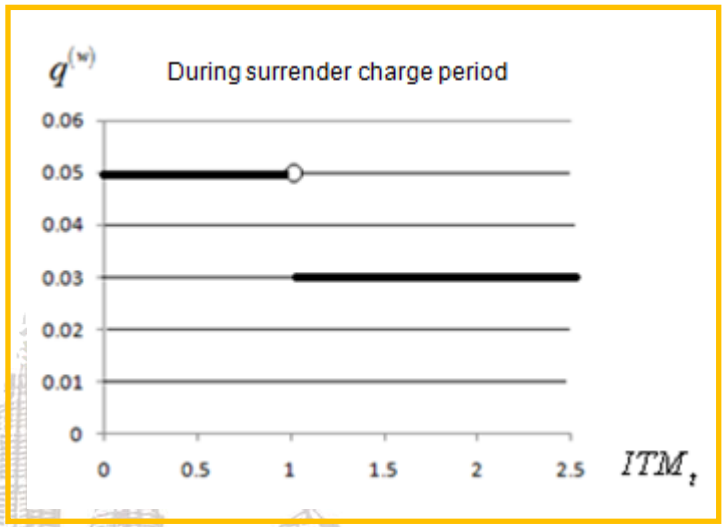
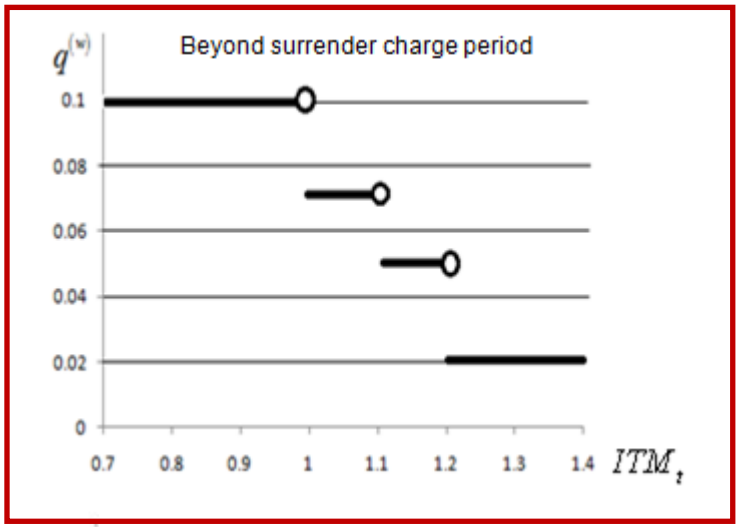
Surrender model (cont.)

- Standard Scenarios of AG 43





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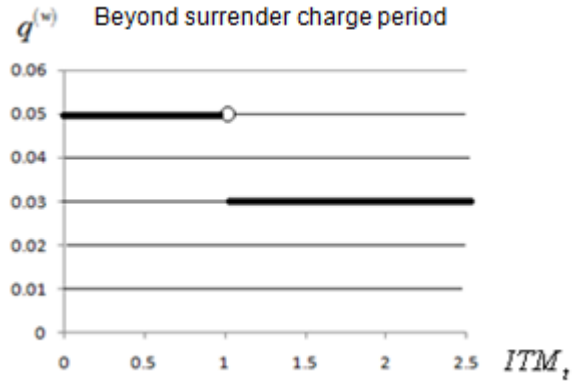
Proposed surrender model

- During Surrender charge period

$$q_t^{(w)} = \begin{cases} 0.05 & \text{if } ITM_t \le 1 \\ 0.03 & \text{if } ITM_t > 1 \end{cases}$$

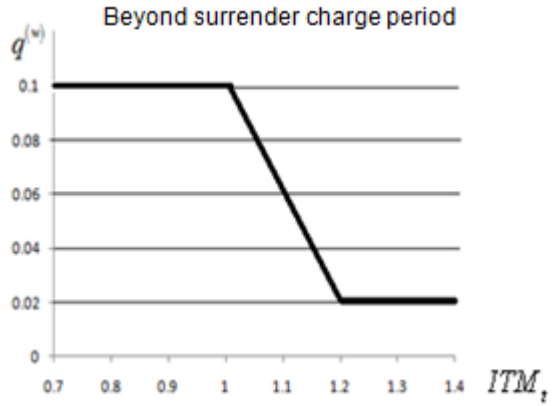


- Beyond surrender charge period

$$q_t^{(w)} = Min\left\{ 0.1 \, , \, Max\left[0.02 \, , \, 0.1 - 0.4 \times (ITM_t - 1) \right] \right\}$$

$$\text{where,} \quad ITM_t = \frac{H_t}{F_t} = \frac{\text{Holding value at time } t}{\text{Account value at time } t}$$

$H_t = ?$



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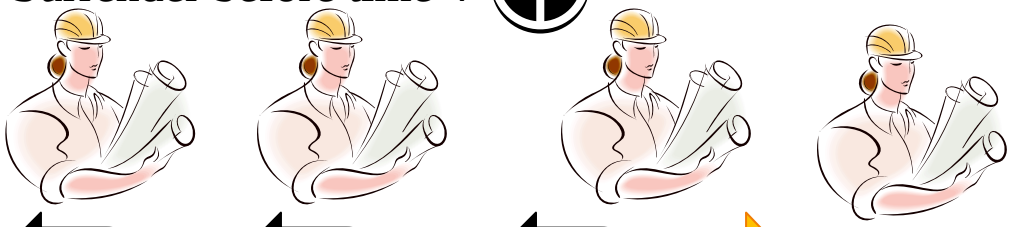
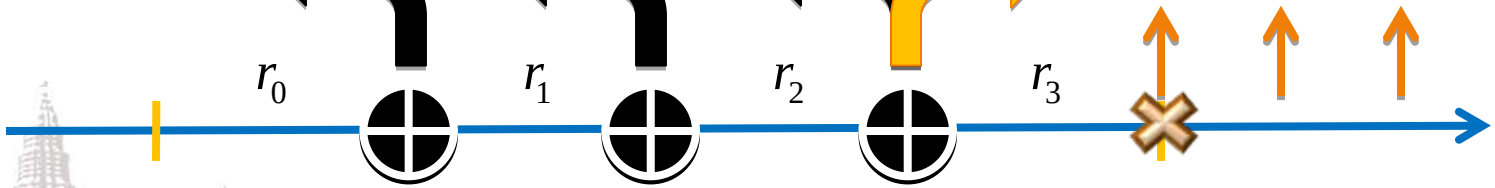
Surrender time

$\frac{H_3}{F_3} \Rightarrow q_3^{(w)}$ *v.s.* ε_3

If $q_3^{(w)} < \varepsilon_3$ Hold tile time 4

If $q_3^{(w)} \geq \varepsilon_3$ Surrender before time 4





time	0	1	2	3	4
age	61	62	63	64	65

Contract value $Max\left(F_4 \times APR \times \ddot{a}_{65}, F_4 \times GAR \times \ddot{a}_{65}\right) = J_4$

Holding value $e^{-r_3}\left[p_{64} \times J_4 + q_{64} \times DB_4\right] = H_3$

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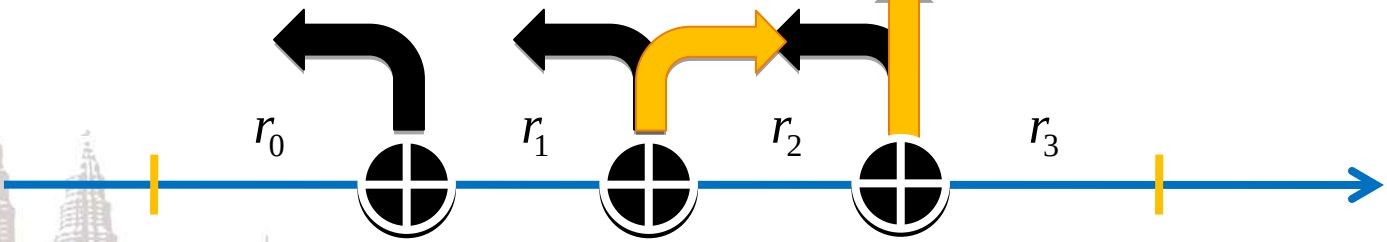
Surrender time (cont.)

$\frac{H_2}{F_2}$ → $q_2^{(w)}$ *v.s.* ε_2

If $q_2^{(w)} < \varepsilon_2$ Hold tile time 3

If $q_2^{(w)} \geq \varepsilon_2$ Surrender before time 3



time	0	1	2	3	4
age	61	62	63	64	65

Contract value $J_3 = F_3$

Holding value $H_2 = e^{-r_2}\left[p_{63} \times J_3 + q_{63} \times DB_3\right]$

Numerical Illustrations

- Assumptions
  - Gender：Male
  - Premium pay：Single premium, monthly premium
  - Accumulation period：10, 15, 20 years
  - Premium
    - Single premium：120,000
    - Monthly premium：

Accumulation period	10	15	20
Premium amount	1,000	666	500

  - Annuity date：the 65th BD
  - Benefit：
    - Surrender during accumulation period, return the account value.
    - With GMDB during accumulation period, death benefit = max(account value, total premium)
    - No GMDB during the accumulation period, death benefit = account value.
    - Choice of APR or GAR at the annuity date.

Numerical Illustrations (cont.)

- Assumptions
  - Life table：1997 Taiwan Insurer Annuity Life Table
  - Fraction age assumption：UDD assumption
  - Standard scenario: $GAR = \frac{1}{9}, T = 10, \sigma_s = 0.3, r_0 = 0.05, q_x = 100\% \times \text{life table}$
  - Interest rate model：Chang（2010）$\hat{\kappa} = 0.014, \hat{\theta} = 0.026, \sigma_r = 0.06$
- Scenarios：
  - GAR: 0.1, 0.111, 0.125
  - Accumulation period (T)：10、15、20
  - Correlation coefficient (ρ)：-0.5, 0, 0.5
  - Asset price volatility(σ_S)： 0.1, 0.3, 0.5
  - Initial interest rate (r_0)：0.025, 0.05, 0.075
  - Mortality：Life table×100%, Life table×90%, Life table×80%

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Single premium - GAO

Charge: 7%

Scenarios		SPA	SPA(s)	Ratio
GAR	1/10	0.0302	0.0293	-3.10%
	1/9	0.0405	0.0394	-2.63%
	1/8	0.0532	0.0522	-1.88%
T	10	0.0405	0.0394	-2.63%
	15	0.0242	0.0237	-2.10%
	20	0.0157	0.0152	-3.21%
ρ	−0.5	0.0405	0.0394	-2.63%
	0	0.0310	0.0303	-2.49%
	0.5	0.0219	0.0213	-2.79%
σ_s	0.1	0.0341	0.0335	-1.86%
	0.3	0.0405	0.0394	-2.63%
	0.5	0.0457	0.0439	-3.87%
r_0	0.025	0.0518	0.0505	-2.57%
	0.05	0.0405	0.0394	-2.63%
	0.075	0.0299	0.0294	-1.58%
死亡率	×1	0.0405	0.0394	-2.63%
	×0.9	0.0446	0.0434	-2.58%
	×0.8	0.0489	0.0477	-2.39%

$$Ratio = \frac{SPA(s) - SPA}{SPA}$$

Ratio

−1.86% ~ −3.87%

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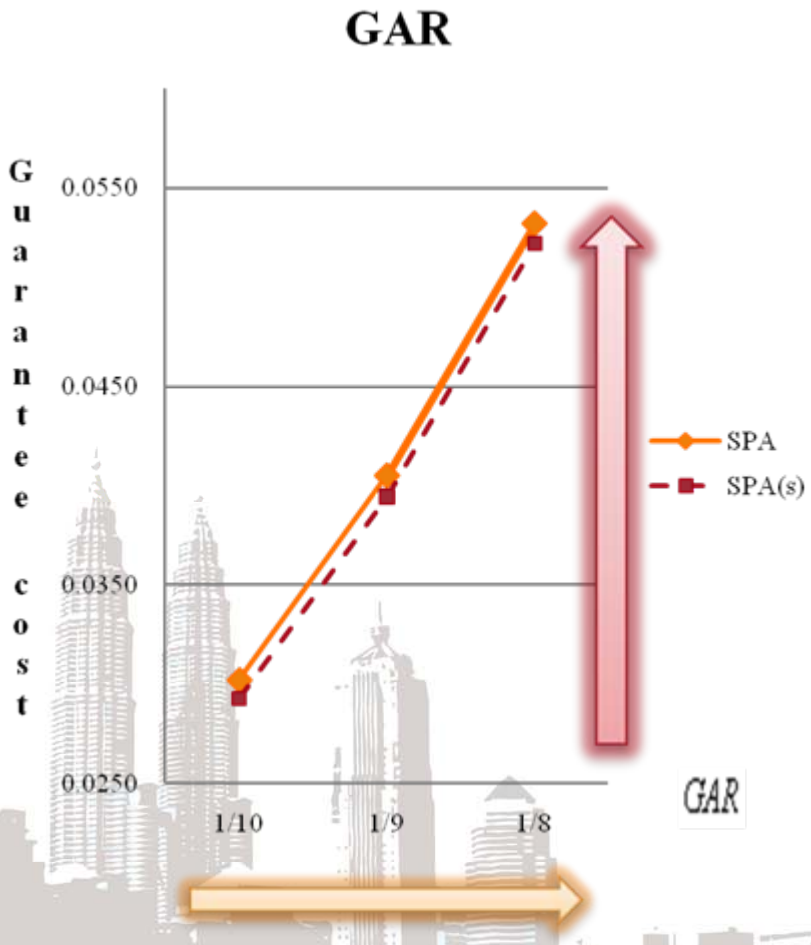
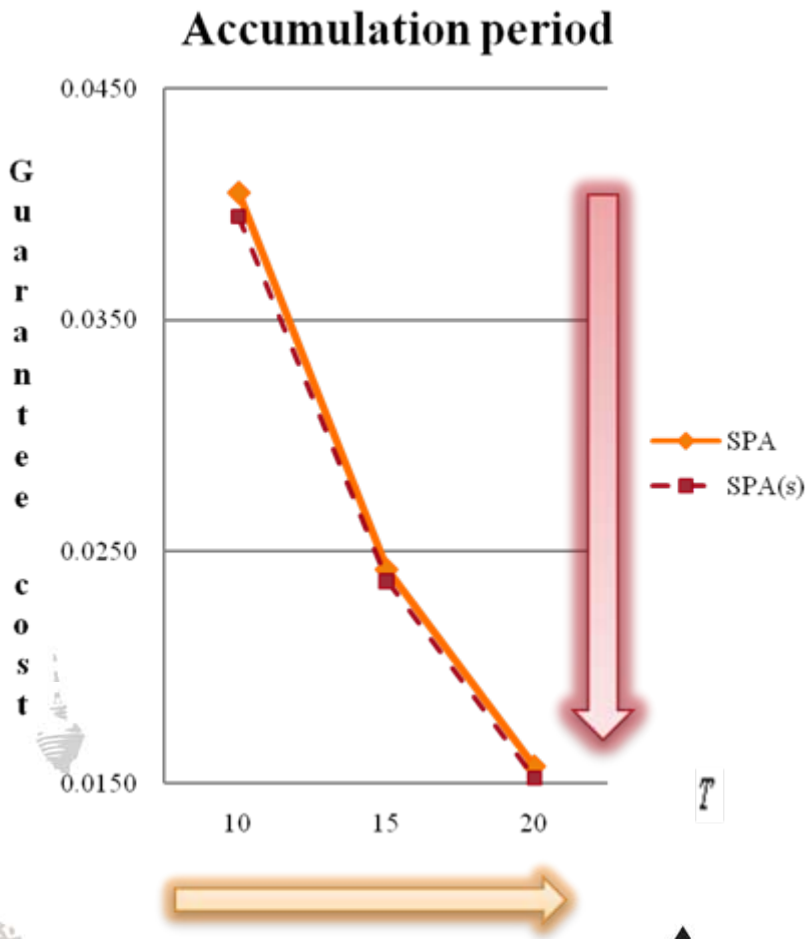
Single Premium - GAO (cont. 1)





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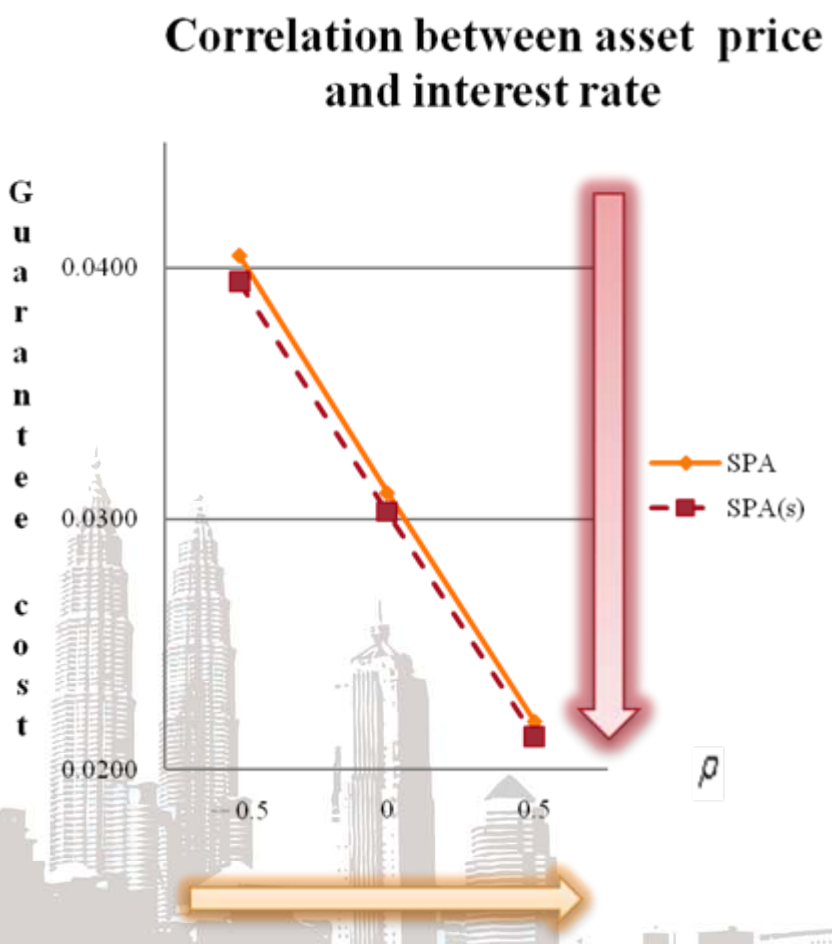
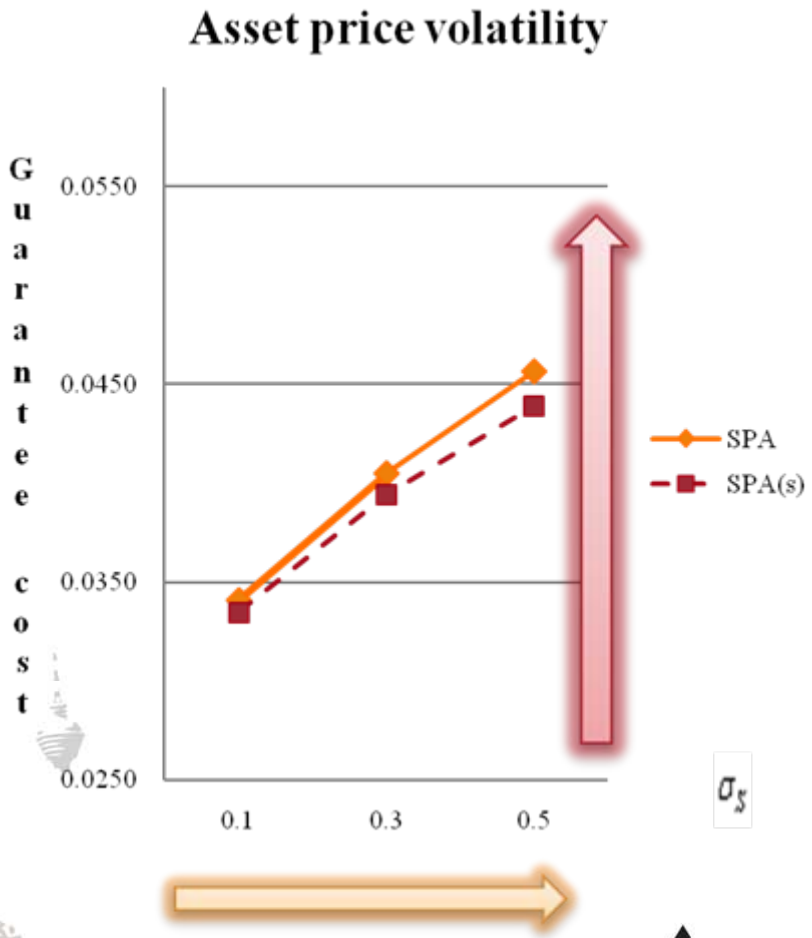
Single Premium - GAO (cont. 2)





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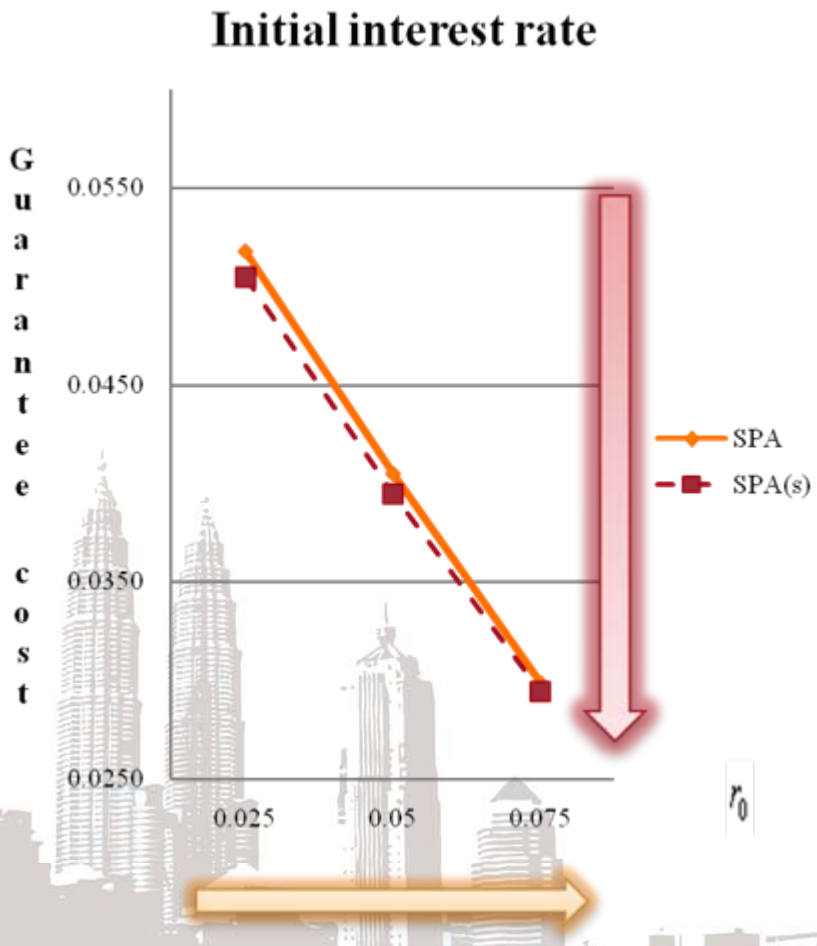
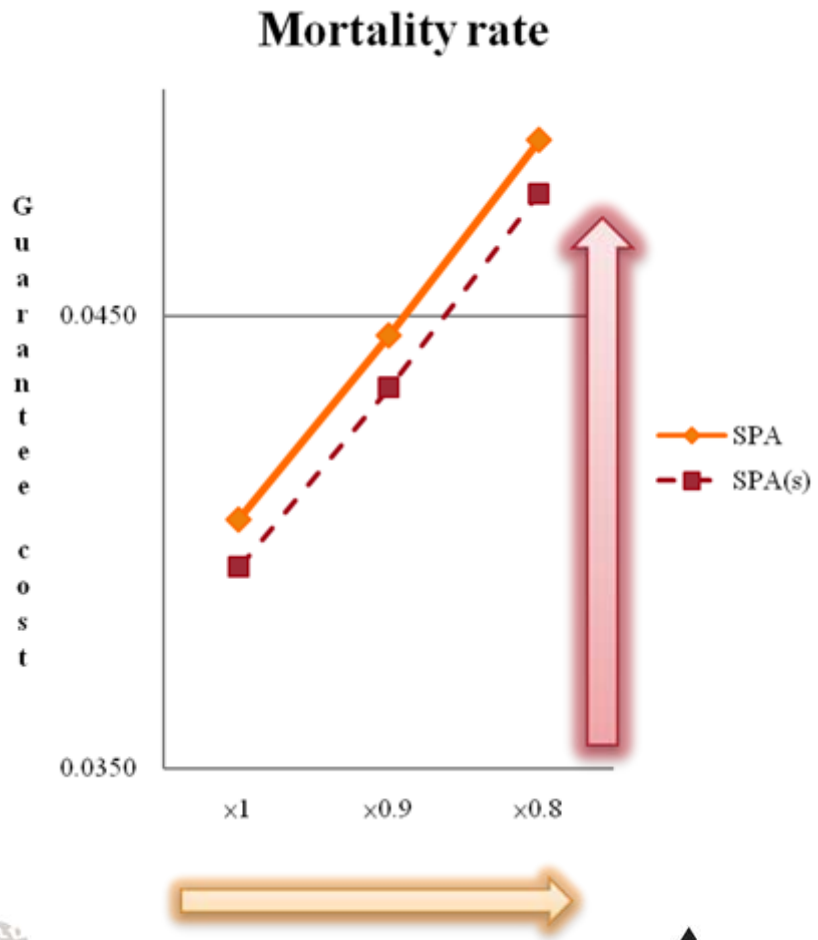
Single Premium - GAO (cont. 3)





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Monthly Premium - GAO

Charge 12%

Scenarios		RPA	RPA(s)	Ratio
GAR	1/10	0.0589	0.0577	-1.98%
	1/9	0.0799	0.0785	-1.80%
	1/8	0.1067	0.1050	-1.58%
T	10	0.0799	0.0785	-1.80%
	15	0.0523	0.0510	-2.57%
	20	0.0373	0.0366	-1.82%
ρ	−0.5	0.0799	0.0785	-1.80%
	0	0.0713	0.0697	-2.19%
	0.5	0.0624	0.0607	-2.70%
σ_s	0.1	0.0742	0.0730	-1.61%
	0.3	0.0799	0.0785	-1.80%
	0.5	0.0865	0.0834	-3.58%
r_0	0.025	0.1118	0.1093	-2.18%
	0.05	0.0799	0.0785	-1.80%
	0.075	0.0554	0.0542	-2.20%
死亡率	×1	0.0799	0.0785	-1.80%
	×0.9	0.0881	0.0866	-1.70%
	×0.8	0.0978	0.0957	-2.22%

Ratio
–1.61% ~ –3.58%

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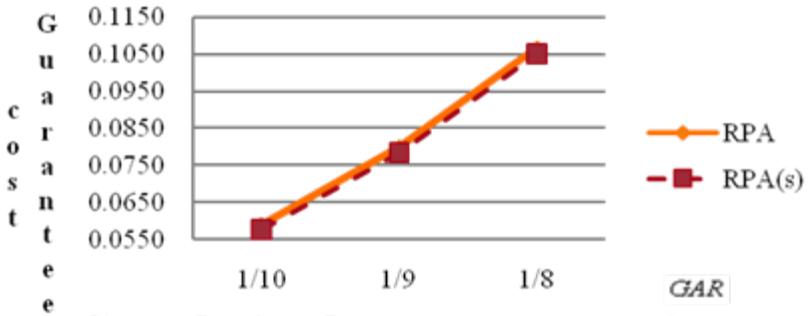
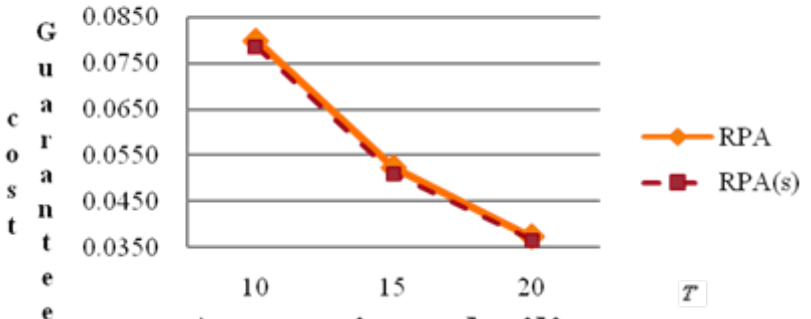
Monthly Premium - GAO (cont.)

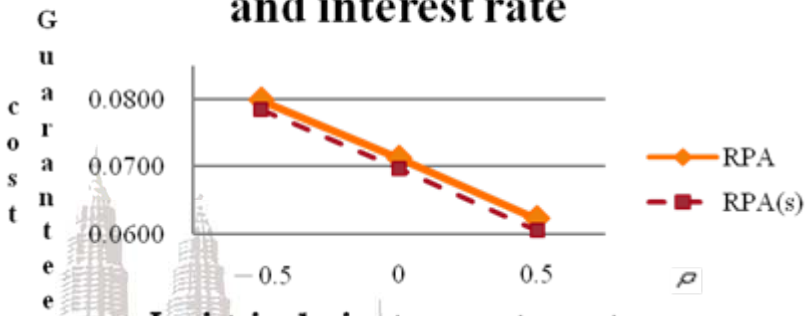
GAR



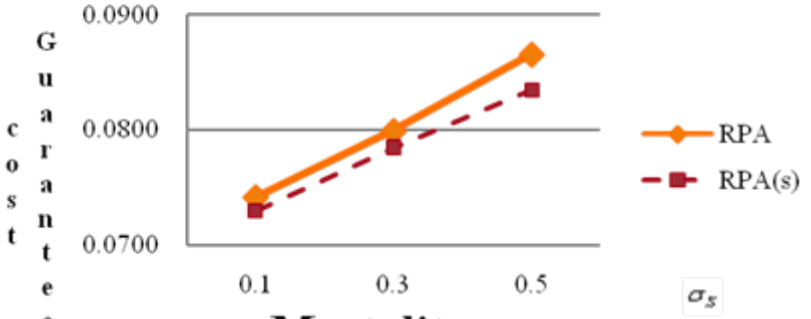
Accumulation period



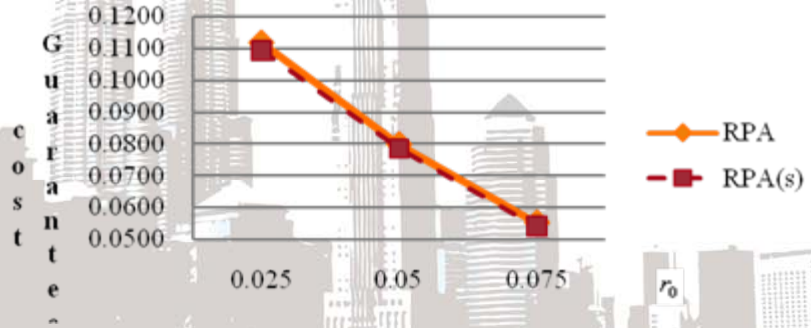
Correlation between asset price and interest rate



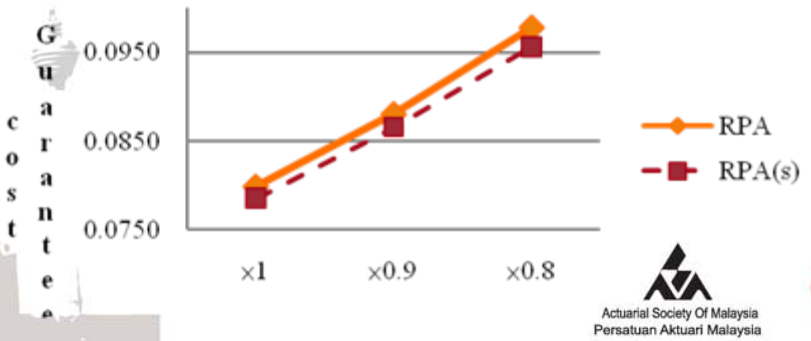
Asset price volatility



Initial interest rate



Mortality



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Single Premium - GAO+GMDB

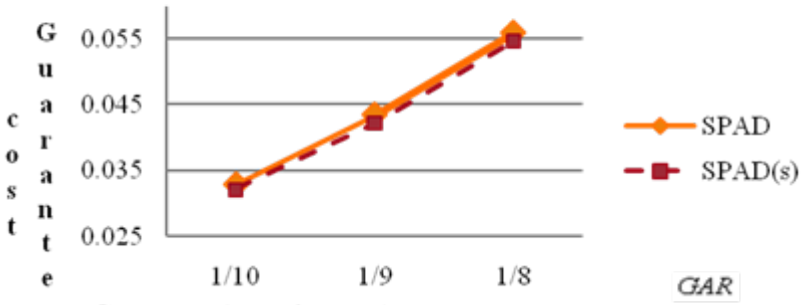
Charge: 7%

Scenarios		SPAD	SPAD(s)	Ratio
GAR	1/10	0.0329	0.0322	-2.15%
	1/9	0.0435	0.0422	-2.91%
	1/8	0.0559	0.0546	-2.31%
T	10	0.0435	0.0422	-2.91%
	15	0.0275	0.0268	-2.56%
	20	0.0189	0.0186	-1.42%
ρ	−0.5	0.0435	0.0422	-2.91%
	0	0.0342	0.0330	-3.44%
	0.5	0.0251	0.0244	-2.74%
σ_s	0.1	0.0356	0.0349	-1.87%
	0.3	0.0435	0.0422	-2.91%
	0.5	0.0504	0.0492	-2.31%
r_0	0.025	0.0557	0.0547	-1.81%
	0.05	0.0435	0.0422	-2.91%
	0.075	0.0320	0.0313	-1.95%
死亡率	×1	0.0435	0.0422	-2.91%
	×0.9	0.0475	0.0454	-4.23%
	×0.8	0.0513	0.0504	-1.69%

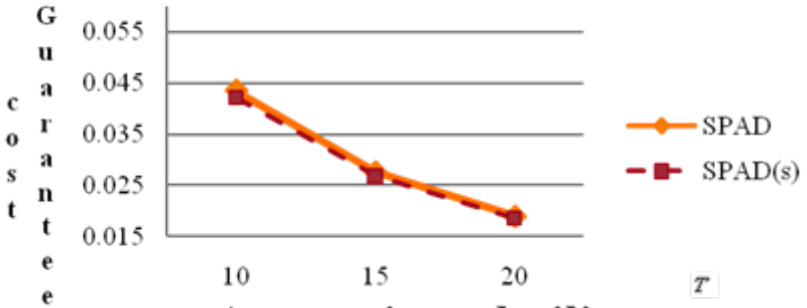
Ratio
–1.42% ~ –4.23%

Single Premium - GAO

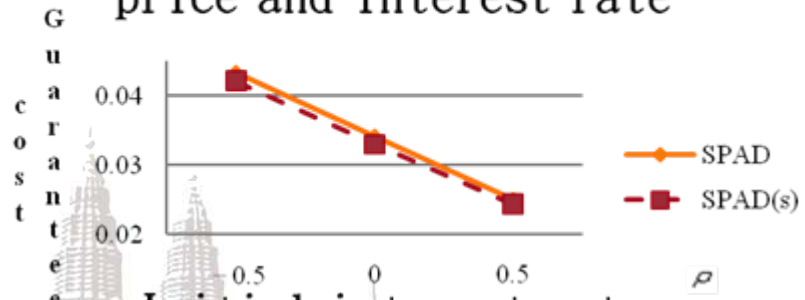
GAR



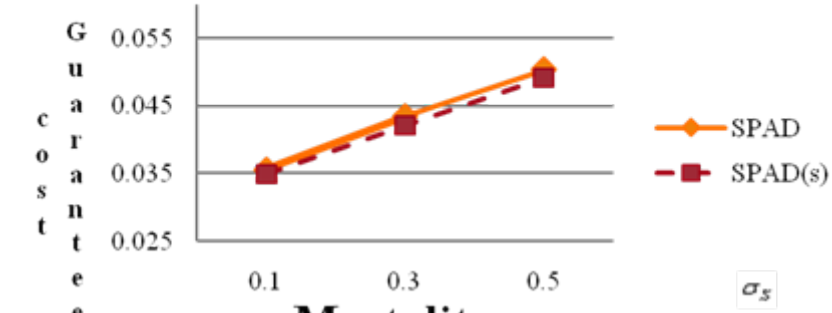
Accumulation period



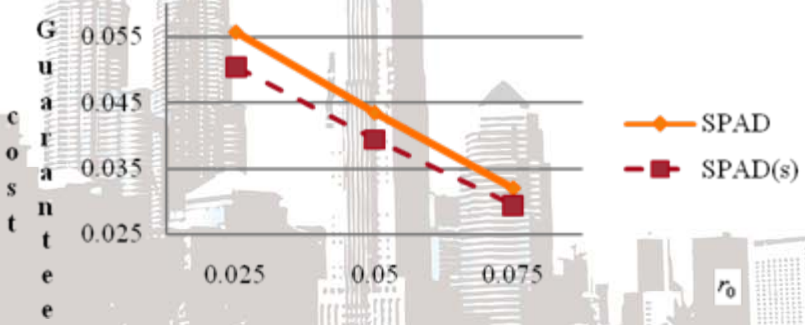
Correlation between asset price and interest rate



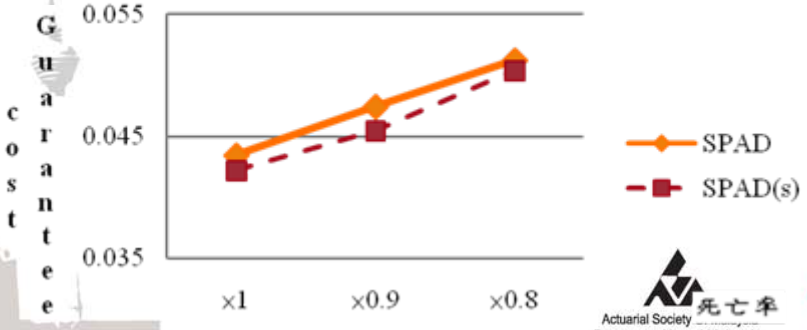
Asset price volatility



Initial interest rate



Mortality



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Monthly premium – GAO+GMDB

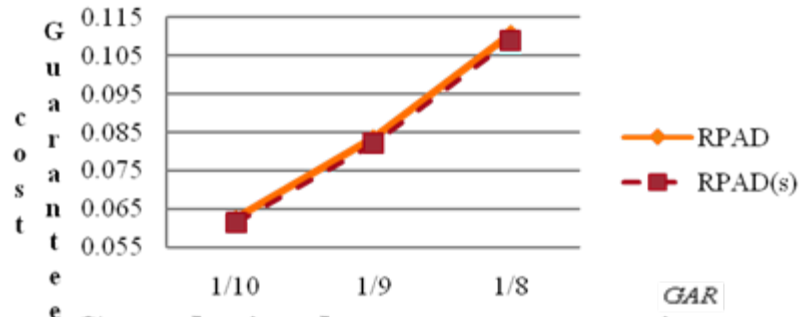
Charge: 12%

Scenarios		RPAD	RPAD(s)	Ratio
GAR	1/10	0.0628	0.0615	-2.12%
	1/9	0.0838	0.0821	-1.93%
	1/8	0.1109	0.1090	-1.76%
T	10	0.0838	0.0821	-1.93%
	15	0.0569	0.0554	-2.64%
	20	0.0427	0.0420	-1.60%
ρ	−0.5	0.0838	0.0821	-1.93%
	0	0.0749	0.0733	-2.18%
	0.5	0.0659	0.0645	-2.12%
σ_s	0.1	0.0770	0.0758	-1.62%
	0.3	0.0838	0.0821	-1.93%
	0.5	0.0914	0.0888	-2.83%
r_0	0.025	0.1164	0.1145	-1.61%
	0.05	0.0838	0.0821	-1.93%
	0.075	0.0585	0.0570	-2.53%
死亡率	×1	0.0838	0.0821	-1.93%
	×0.9	0.0916	0.0894	-2.39%
	×0.8	0.1000	0.0984	-1.62%

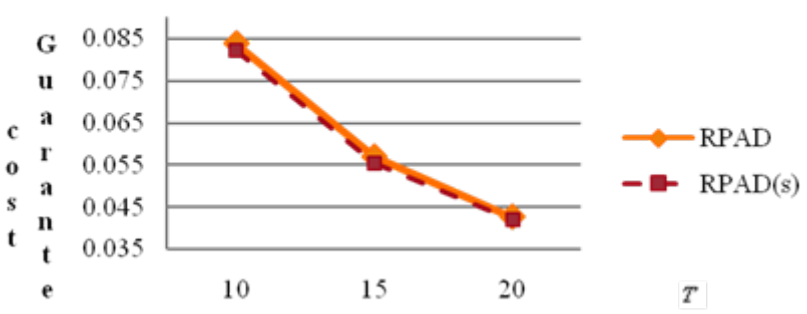
Ratio
–1.60% ~ –2.83%

Monthly premium – GAO+GMDB(Cont.)

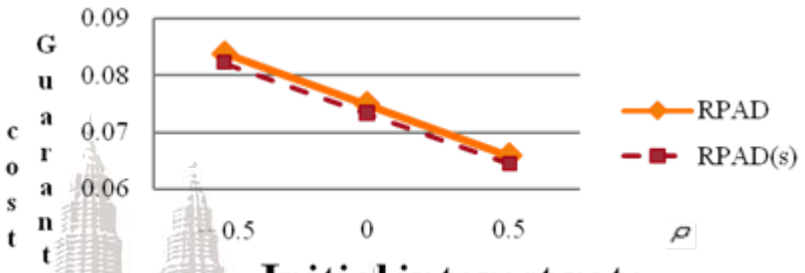
GAR



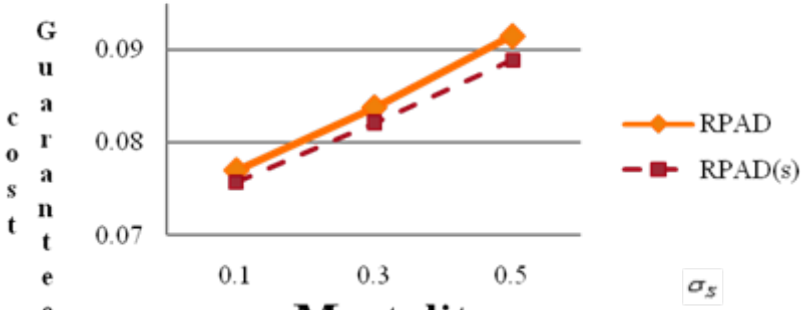
Accumulation period



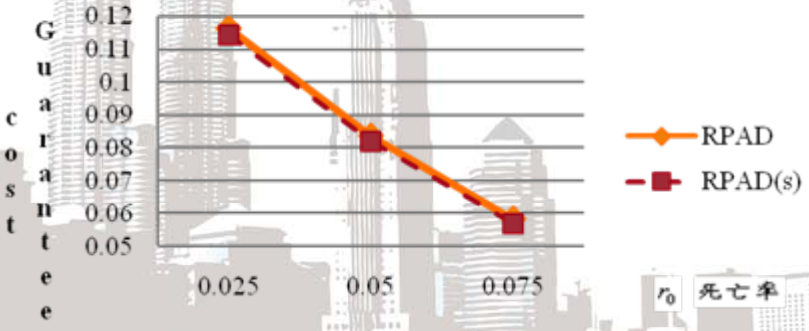
Correlation between asset price and interest rate



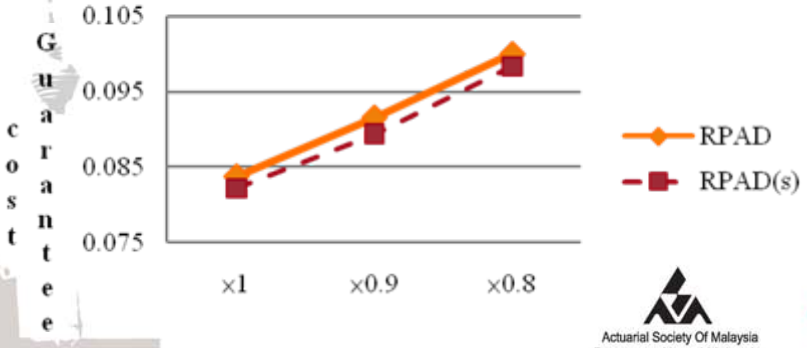
Asset price volatility



Initial interest rate



Mortality



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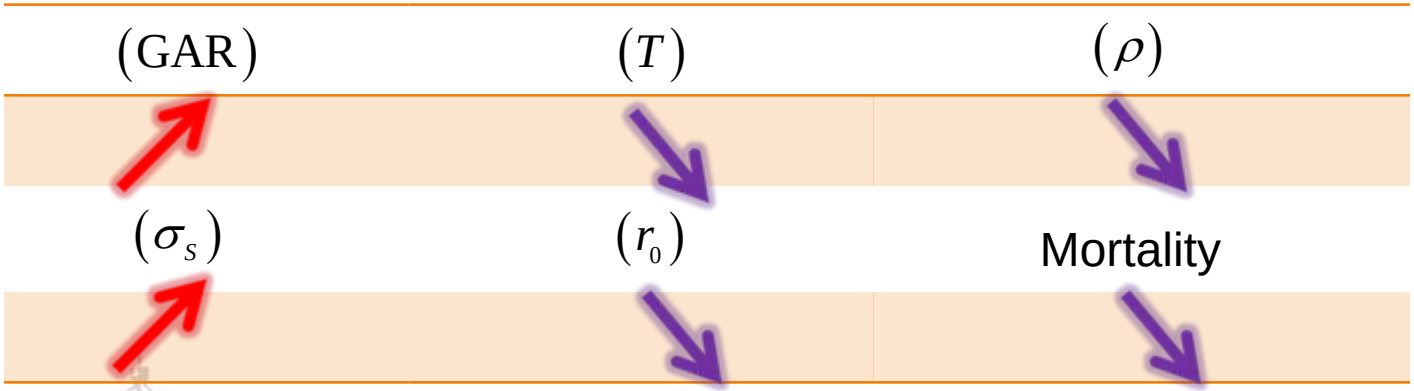
Summary

- Single premium or monthly premium, GAO or GAO+GMDB
  - Guarantee cost m_C ↗



  - Guarantee cost reduced with our surrender model

Thank you for your listening !

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