

Basic Ratemaking

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- **Academy**
 - Fu-Jen Catholic University in Applied Math
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 - CIGNA Life Insurance Company - Taiwan Branch, 1 yr
 - Ming Tai Fire & Marine Insurance Company, 1 yr
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 - St. Paul Travelers Company, 6.0 yrs
 - Technology Actuarial, Associate Actuary
 - Personal Line Actuarial, Actuary
 - Zurich Insurance Company, Taiwan, 1 yr
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 - KPMG, Taiwan
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 - Fellow of Casualty Actuarial Society (FCAS), 2005
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Basic Ratemaking

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- Chapters
 - Chapter 1 : Introduction
 - Chapter 2 : Rating Manuals
 - Chapter 3 : Ratemaking Data
 - Chapter 4 : Exposures
 - Chapter 5 : Premium
 - Chapter 6 : Losses and LAE
 - Chapter 7 : Other Expenses and Profit
 - Chapter 8 : Overall Indication

Chapter 1 Introduction



Chapter 1 Introduction - Basic Insurance Terms

- **Price = Cost + Profit**
 - For many non-insurance goods and services, the production cost is known before the product is sold
 - For insurance, the ultimate cost of an insurance policy is not known at the time of the sale
- **Exposure**
 - The basic unit of risk that underlies the insurance premium
- [Note] The premium is calculated based on a given rate per unit of risk exposed
- **Written Exposure**
 - The total exposure arising from policies issued during a specific period of time, such as a calendar year or quarter
- **Earned Exposure**
 - The portion of the written exposure for which coverage has already been provided as of a certain point of time

Chapter 1 Introduction - Basic Insurance Terms

- **Unearned Exposure**
 - The portion of the written exposures for which coverage has not yet been provided as of that point in time
- **In-Force Exposure**
 - The number of insured units that are exposed to loss at a given point in time
- Will be discussed further in Chapter 4
- **Premium**
 - The amount the insured pays for insurance coverage
 - Same concepts of written, earned, unearned, and in-force as those used in exposure
 - Will be discussed further in Chapter 5

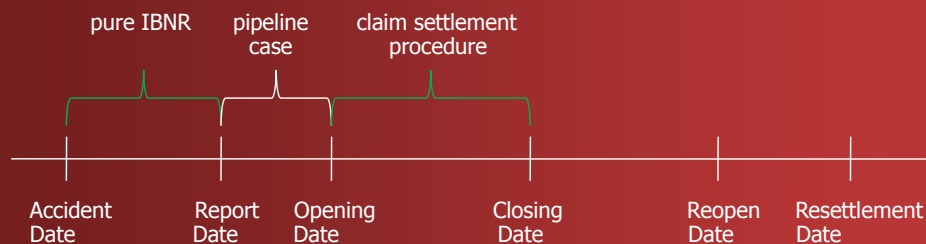
Chapter 1 Introduction - Basic Insurance Terms

■ Claim

- The demand that an insured makes to the insurer for indemnification under an insurance policy

[Note] The individual making a claim is called a claimant

- Various dates associated with a claim



Chapter 1 Introduction - Basic Insurance Terms

■ Loss

- The amount of compensation paid or payable to the claimant

- Paid Loss

- Those amounts that have been paid to claimants

- Case Reserve

- An estimate of the amount of money required to ultimately settle that claim

[Note] The case reserve excludes any payments already made

- Reported Loss (or Case Incurred Loss)

- Paid Loss + Case Reserve

Chapter 1 - Introduction

- Ultimate Loss
 - The amount of money required to close and settle all claims for a defined group of policies
 - The aggregate sum of reported losses across all known claims may not equal the ultimate loss for many years
 - The reported losses on existing claims may change over time
 - At any point in time, there may be unreported claims
 - IBNR Reserve (Incurred but Not Reported)
 - The amount estimated to ultimately settle the unreported claims
 - Usually refer to pure IBNR
 - IBNER Reserve (Incurred but Not Enough Reported)
 - The difference between the aggregate reported losses at the time the losses are evaluated and the aggregate amount estimated to ultimately settle these reported claims
 - a.k.a. Development on Known Claims or Case Emergence
- [Note] Broad IBNR includes pure IBNR and IBNER and all other reserves, but not case reserve
- [Note] Estimate Ultimate Loss = Reported Loss + IBNR + IBNER

Chapter 1 Introduction - Basic Insurance Terms

- Loss Adjustment Expense (LAE)
 - The expense occurred by an insurer to close a claim
 - ALAE (Allocated Loss Adjustment Expense)
 - Claim-related expenses that are directly attributable to a specific claim
 - Example, fees associated with outside legal counsel hired to defend a claim can be directly assigned to a specific claim
 - ULAE (Unallocated Loss Adjustment Expense)
 - Claim-related expenses that cannot be directly assigned to a specific claim
 - Example, salaries of claims department

Chapter 1 Introduction - Basic Insurance Terms

- **Underwriting Expenses**
 - The expenses for acquiring and servicing the policies
 - Commission and Brokerage
 - Other Acquisition
 - General Expense
 - Taxes, Licenses, and Fees
 - Commission and Brokerage
 - Amount paid to insurance agents or brokers as compensation for generating business
 - Typically, paid as a percentage of premium written
 - Other Acquisition
 - Expenses other than commissions and brokerage expense paid to acquire business
 - Example, advertisements
 - General Expense
 - The remaining expenses associated with the insurance operations and any other miscellaneous costs
 - Taxes, Licenses, and Fees
 - All taxes and miscellaneous fees paid, but excluding income taxes

Chapter 1 Introduction – Fundamental Insurance Equation

- **Fundamental Insurance Equation**
 - $\text{Price} = \text{Cost} + \text{Profit}$
 - $\text{Premium} = \text{Losses} + \text{LAE} + \text{UW Expenses} + \text{UW Profit}$
 - The goal of ratemaking is to assure that the fundamental insurance equation is appropriately balanced
 - Rates should be set so that the premium is expected to cover all costs and achieve the target underwriting profit
 - Two key points to consider
 - Ratemaking is prospective
 - Balance should be attained at the aggregate and individual levels

Chapter 1 Introduction – Fundamental Insurance Equation

- **Ratemaking is prospective**
 - Insurance is a promise to provide compensation in the event a specific loss event occurs during a defined period in the future
 - Using relevant historical experience to estimate the future expected costs
 - [Note] This doesn't mean actuaries are setting premium to recoup past losses
 - It is important to recognize that adjustments will be necessary to convert the experience into that which will be expected in the future when the rates will be in effect
 - Rate changes
 - Operational changes
 - Inflationary pressures
 - Law changes



Chapter 1 Introduction – Fundamental Insurance Equation

- **Overall and individual balance**
 - Overall level
 - Ensures that the total premium for all policies is sufficient to cover the total expected losses and expenses and to provide for the targeted profit
 - Individual level
 - A policy that presents significantly higher risk of loss should have a higher premium than a policy that represents a significantly lower risk of loss

Chapter 1 Introduction - Basic Insurance Ratios

■ Frequency

- A measure of the rate at which claims occur
- Frequency = # of Claims / # of Exposures
 - [Note] Earned Exposure should be used for the denominator
- Help measure the effectiveness of specific underwriting actions

[Example 1-1] If there are 100,000 written exposures, and 80% of the written exposures are earned during 2010. What is the frequency for the year 2009 if 40,000 claims occurred during the year?

[Sol] Earned Exposure = 80,000

$$\text{Frequency} = 40,000 / 80,000 = 0.5$$

Chapter 1 Introduction - Basic Insurance Ratios

■ Severity

- A measure of the average cost of claims
- Severity = Total Losses / # of Claims
- Can be
 - Paid Severity = Paid Losses / # of Claims Closed
 - Reported Severity = Reported Losses / # of Claims Reported
- Help measure the loss trends

[Example 1-2] Continue with Example 1-1, if the total losses are \$200,000 thousand for the 40,000 claims. What is the severity?

[Sol] Severity = \$200,000 / 40,000 = \$5 thousand

Chapter 1 Introduction - Basic Insurance Ratios

■ Pure Premium (PP)

- As known as Loss Cost, or Burning Cost
- A measure of the average loss per exposure
- $PP = \text{Total Losses} / \# \text{ of Exposures}$
= Frequency x Severity

[Note] Ultimate losses and earned exposures should be used

- Help measure the overall loss cost trends

[Example 1-3] Continue with Example 1-2, please determine the pure premium.

[Sol] $PP = \$200,000 / 80,000 = 0.5 \times \$5 \text{ thousand}$
= \$2.5 thousand

Chapter 1 Introduction - Basic Insurance Ratios

■ Average Premium (PP)

- Average Premium = Total Premium / # of Exposures

[Note] Premium and exposures should be on the same basis (both written or earned)

- Changes in average premium may indicate
 - Rate change
 - Change in the mix of business, e.g. deductible shift, insured limit shift, risk shifts, and etc

[Example 1-4] Continue with Example 1-1, please determine the average written premium if the written premium is \$300,000 thousand.

[Sol] Average Premium = $\$300,000 / 100,000 = \$3$ thousand

Chapter 1 Introduction - Basic Insurance Ratios

■ Loss Ratio (LR)

- A measure of the portion of each premium dollar used to pay losses
- $LR = \text{Total Losses} / \text{Total Premium}$
 $= \text{Pure Premium} / \text{Average Premium}$
 - [Note] Ultimate losses and earned premium should be used
- Help measure the adequacy of the rates

[Example 1-5] Continue with the examples above, assuming the written premium is earned at the same proportion as written exposure, Please determine the loss ratio.

[Sol] Earned Premium = \$300,000 × 80% = \$240,000
 $LR = \$200,000 / \$240,000 = \$2.5 / \$3 = 83.33\%$

Chapter 1 Introduction - Basic Insurance Ratios

■ Loss Adjustment Expense Ratio (LAE Ratio)

- $LAE \text{ Ratio} = \text{Total LAE} / \text{Total Losses}$

[Note] LAE includes both ALAE and ULAE

■ Underwriting Expense Ratio (UW Expense Ratio)

- A measure of the portion of each premium dollar used to pay for underwriting expenses
- $UW \text{ Expense Ratio} = \text{Total UW Expenses} / \text{Total Premium}$
- Two categories

- Variable expense, e.g. commissions, licenses and fees
- Fixed expense, e.g. salaries, office rental

[Note] The written premium is used to measure the variable expense ratio, and the earned premium is used to measure the fixed expense ratio

Chapter 1 Introduction - Basic Insurance Ratios

- **Operating Expense Ratio (OER)**
 - A measure of the portion of each premium dollar used to pay for loss adjustment and underwriting expense
 - $OER = \frac{UW \text{ Expense Ratio} + \text{Total LAE}}{\text{Total Earned Premium}}$
 - [Note] LAE includes both ALAE and ULAE
 - Used to monitor operational expenditures
- **Combined Ratio**
 - A primary measure of the profitability of the book of business
 - $\text{Combined Ratio} = \text{Loss Ratio} + OER$
 - [Note] Loss ratio should not include LAE

Chapter 1 Introduction - Basic Insurance Ratios

- **Renewal Ratio**
 - Retention ratio is used in this book
 - A measure of the rate at which existing insureds renew their policies upon expiration
 - $\text{Renewal Ratio} = \frac{\# \text{ of Policies Renewed}}{\# \text{ of Potential Renewal Policies}}$
- **Close Ratio**
 - A measure of the rate at which prospective insureds accept a new business quote
 - $\text{Close Ratio} = \frac{\# \text{ of Accepted Quote}}{\# \text{ of Quotes}}$
 - Is used to determine the competitiveness of rates for new business

Chapter 2 Rating Manuals



Chapter 2 Rating Manuals

- Rating manual usually includes
 - Rules
 - Rate pages
 - Rating algorithm
 - Underwriting guidelines

Chapter 2 Rating Manuals – Rules

■ Rules

- Typically contain qualitative information that is needed to understand and apply the quantitative rating algorithms
 - For example, a primary residence may be defined in the rule for a homeowner policy
- To be an aid in calculating premium

Chapter 2 Rating Manuals – Rate Pages

■ Rate pages usually include

- Base rates
- Rating tables
- Fees

■ Base Risk

- A specific risk pre-defined by the insurer
- Represents a set of risk characteristics that are most common, or target market

■ Base Rate

- The rate applied to the base risk
- Not the average rate
- The rate for risks other than base risk is determined by modifying the base rate by a series of multipliers or addends or some unique mathematical expression

Chapter 2 Rating Manuals – Rating Algorithms

- Describes in detail how to combine the various components in the rules and rate pages to calculate the premium charged for any risk
- May include
 - The order in which rating variables should be considered
 - Multiplicative or additive
 - Maximum and minimum premium

Chapter 2 Rating Manuals – Underwriting Guidelines

- Underwriting guidelines may be used to specify
 - Decisions to accept, decline, or refer (to senior underwriter) risks
 - Company placement
 - Schedule rating credits/debits

Chapter 2 Rating Manuals – Example : Residential Fire Insurance

■ Rules

- 使用性質種類定義:

使用性質種類	說明
1. 住宅	家庭手工副業，仍照住宅 費率計費。
2. 公共宿舍	一、附著建物，雖為各別 屋頂，仍適用本項費率。 二、全部單身宿舍，雖有 十人以上而不各別舉炊者，不以公共宿舍論。 三、一部分為眷屬宿舍， 一部分為單身宿舍者，仍應按間併入計算居住家數。
3. 連幢住宅	住宅區之連幢住宅十一間以上者，按本項費率計費，建築等級以該一連接 建築之外牆為準。倘其中 有行號商店或其他使用性質，仍應併入計算間數。 至各該行號商店等，仍應 按其本身使用性質單獨計費，如各別費率低於本項費率時，並仍按本項費率 計費。

- 公寓式樓房、非公寓式樓房、高樓大廈定義
- 自動滅火設備定義

Chapter 2 Rating Manuals – Example : Residential Fire Insurance

■ Rate pages

- Base Rate

使用性質種類	特一等	特二等	頭等	二等	三等
1. 住宅	0.141	0.167	0.202	0.312	0.412
2. 公共宿舍	0.267	0.320	0.384	0.591	0.784
3. 連幢住宅	0.362	0.438	0.522	0.808	1.070

- 高樓加費

建築物樓層別	加費比率
15~24層樓	10%
25層樓(含)以上	15%

Chapter 2 Rating Manuals – Example : Residential Fire Insurance

– 營業加費

1. 公寓式樓房	同層或他層(含地下層)有營業行為者，住宅按其本身基本費率加25%營業加費。
2. 非公寓樓房	同層或他層(含地下層)有營業行為者，住宅按其本身基本費率加25%營業加費。
3. 高樓大廈	同層或他層(含地下層)有營業行為者，住宅按其本身基本費率加50%營業加費。

– 消防設備減費

減費項目		減費比率
火警自動警報設備	建築物備有火警自動警報設備	5%
室內消防栓	建築物備有室內消防栓設備	5%
自動滅火設備	整棟建築物之有效防護範圍超過建築物總面積90%者。	A、各層樓之有效防護範圍達該樓層面積100%者
		B、各層樓之有效防護範圍達該樓層面積90%以上者
		C、各層樓之有效防護範圍達該樓層面積100%者
	整棟建築物之有效防護範圍未達建築物總面積90%者。	10%

Chapter 2 Rating Manuals – Example : Residential Fire Insurance

– 附加費用率

- 保險代理人：44.5%
- 保險經紀人：44.5%
- 保險業務員：44.5%
- 直接業務(要保人直接採購或投保)：33.3%
- 其他通路：44.5%

■ Rating algorithm

- $\text{Total Premium} = \text{Amount of Insurance} \times \text{Base Rate} \times (1 + \text{高樓加費} + \text{營業加費} - \text{消防設備減費}) / (1 - \text{附加費用率})$ 。

Chapter 3

Ratemaking Data



Chapter 3 Ratemaking Data – Internal Data

- Two types
 - Risk information
 - Exposures
 - Premium,
 - Claim counts
 - Losses
 - Accounting information
 - Underwriting expense
 - ULAE

Chapter 3 Ratemaking Data – Risk Data

- Two databases
 - Policy Database
 - Claim Database
 - Policy Database
 - Policy identifier
 - Risk identifier
 - One policy may have multiple risks, e.g. two cars, two locations of insured
 - Relevant dates
 - Original effective date
 - Original expiration date
 - Date of amendment
 - Premium
 - Exposure
 - Characteristics
- [Note] If there is midterm adjustments, the characteristics corresponding to each adjustment must be recorded

Chapter 3 Ratemaking Data – Risk Data

[Example 3-1]

- Policy A is written on 1/1/2010, with an annual premium of \$1,100. The home is located in Territory 1 and the insured has a \$250 deductible. The policy remains unchanged for the full term of the policy
- Policy B is written on 4/1/2010, with an annual premium of \$600. The home is located in Territory 2 and the insured has a \$250 deductible. The policy is canceled on 12/31/2010
- Policy C is written on 7/1/2010, with an annual premium of \$1,000. The home is located in Territory 3 and the insured has a \$500 deductible. On 1/1/2011, the insured decrease the deductible to \$250. The full annual term premium after the deductible change is \$1,200

Chapter 3 Ratemaking Data – Risk Data

[Sol]

Policy Database

Policy	Ori. Eff. Date	Ori. Exp. Date	Transaction Eff. Date	Ded.	Terr	Written Exposure	WP
A	1/1/2010	12/31/2010	1/1/2010	250	1	1	1,100
B	4/1/2010	3/31/2011	4/1/2010	250	2	1	600
B	4/1/2010	3/31/2011	12/31/2010	250	2	-0.25	-150
C	7/1/2010	6/30/2011	7/1/2010	500	3	1	1,000
C	7/1/2010	6/30/2011	1/1/2011	500	3	-0.5	-500
C	7/1/2010	6/30/2011	1/1/2011	250	3	0.5	600

Chapter 3 Ratemaking Data – Risk Data

- Claim Database
 - Policy identifier
 - Risk identifier
 - Claim identifier
 - Claimant identifier
 - Event identifier
 - Relevant loss dates
 - Report date
 - Date of loss payment
 - Date of reserve change
 - Date of claim status change
 - Claim status
 - Open, Closed
 - Reopen, Reclosed

Chapter 3 Ratemaking Data – Risk Data

- Paid loss
- Case reserve
- ALAE
 - ULAE is handled elsewhere
- Salvage & Subrogation
- Type of injury
- Cause of loss

Chapter 3 Ratemaking Data – Risk Data

[Example 3-2]

- Policy A: A covered loss occurs on 1/10/2010. The claim is reported to the insurance company on 1/15/2010, and an initial case reserve of \$10,000 is established. An initial payment of \$1,000 is made on 3/1/2010, with a corresponding \$1,000 reduction in the case reserve. A final payment of \$9,000 is made on 5/1/2010, and the claim is closed
- Policy C: A covered loss occurs on 10/1/2010, is reported on 10/15/2010, and a case reserve of \$18,000 is established. The insurer makes a payment of \$2,000 on 12/15/2010, and reduces the case reserve to \$17,000. An additional payment of \$7,000 is made on 3/1/2011, and the case reserve is reduced to \$15,000. The claim is closed on 3/1/2012, when the insurer makes a final payment of \$15,000 and receives a \$1,000 salvage recovery by selling damaged property
- Policy C: A second loss occurs on 2/1/2011. The claim is reported on 2/15/2011, and an initial reserve of \$15,000 is set. On 12/1/2011, the company pays a law firm \$1,000 for fees related to handling of the claim. The claim is closed on that date with no loss payments made

Chapter 3 Ratemaking Data – Risk Data

[Sol]

Claim Database

Policy	Claim Number	Accident Date	Report Date	Trans. Date	Claim Status	Paid Loss	Case Reserve	Paid ALAE	S&S
A	1	1/10/2010	1/15/2010	1/15/2010	Open	0	10,000	0	0
A	1	1/10/2010	1/15/2010	3/1/2010	Open	1,000	9,000	0	0
A	1	1/10/2010	1/15/2010	5/1/2010	Closed	9,000	0	0	0
C	2	10/1/2010	10/15/2010	10/15/2010	Open	0	18,000	0	0
C	2	10/1/2010	10/15/2010	12/15/2010	Open	2,000	17,000	0	0
C	2	10/1/2010	10/15/2010	3/1/2011	Open	7,000	15,000	0	0
C	2	10/1/2010	10/15/2010	3/1/2012	Closed	15,000	0	0	1,000
C	3	2/1/2011	2/15/2011	2/15/2011	Open	0	15,000	0	0
C	3	2/1/2011	2/15/2011	12/1/2011	Closed	0	0	1,000	0

Chapter 3 Ratemaking Data – Accounting Information

- Underwriting Expenses
 - Expenses incurred in the acquisition and servicing of the policies
 - Includes
 - General expenses
 - Other acquisition expenses
 - Commissions and brokerage
 - Taxes, licenses, and fees
- ULAE

Chapter 3 Ratemaking Data – Data Aggregation

- 4 ways to aggregate data for different types of analyses
 - Calendar year (CY)
 - Accident year (AY)
 - Policy year (PY)
 - Report year (RY)
- Calendar Year
 - Consider all premium and loss transactions that occur during the 12 month calendar year
 - All premium and exposures are fixed at the end of the CY
 - CY data is available quickly once the CY ends
 - Main disadvantage of CY is the mismatch in timing between premium and losses
 - Premium earned during CY come from policies in force during that year
 - Losses may include payments and reserve changes on claims from policies issued years ago

Chapter 3 Ratemaking Data – Data Aggregation

- Accident Year
 - Considers losses for accidents that have occurred during a 12-month period, regardless of the policy issued date
 - Premium and exposures are defined as the same as CY aggregation
 - Reported losses consist of loss payments made plus case reserves only for those claims that occurred during that year
 - Reported losses can and often do change at the end of AY
 - Additional claims are reported and paid
 - Reserves are changed
 - Future development on those known losses needs to be estimated
 - Better match of premium and losses than CY
 - Losses on accidents occurring during the year are compared to premium earned on policies during the same year

Chapter 3 Ratemaking Data – Data Aggregation

- **Policy Year**
 - Considers all premium and loss transaction on policies that were effective during a 12-month period
 - Premium and exposures are not fixed until after the expiration date
 - Reported losses for PY consist of payments made plus case reserves only for those claims covered by policies effective during the year
 - Reported losses can and often do change at the end of AY
 - Additional claims are reported and paid
 - Reserves are changed
 - Best match between losses and premium
 - Losses on policies effective during the year are compared with premium earned on those same policies
 - Data takes longer to develop than both CY and AY
 - For a product with an annual policy term, premium are not fully earned until 24 months after the start of the policy year

Chapter 3 Ratemaking Data – Data Aggregation

- **Report Year**
 - Considers losses for accidents that are reported during a 12-month period, regardless when the claim occurred
 - Premium and exposures are defined as the same as CY aggregation
 - Reported losses consist of loss payments made plus case reserves only for those claims that are reported during that year
 - Reported losses can and often do change at the end of AY
 - Additional claims are reported and paid
 - Reserves are changed

Chapter 4 Exposures



Chapter 4 Exposures – Exposures

- The basic unit that measures a policy's exposure to loss
- Serves as the basis for the calculation of premium
 - Base rates are typically expressed as a rate per exposure

Chapter 4 Exposures – Criteria for Exposure Bases

- A good exposure base should meet the following three criteria
 - Directly proportional to expected loss
 - Practical
 - Consider any preexisting exposure base established within the industry
- Proportional to Expected Loss
 - All else being equal, the expected loss of a policy with two exposures should be twice the expected loss of a similar policy with one exposure
 - Doesn't mean that the exposure base is the only item by which losses may systematically vary
 - Should be responsive to any change in exposure to risk

Chapter 4 Exposures – Criteria for Exposure Bases

- Example, for homeowners insurance, number of house years vs. amount of insurance
- Practical
 - Should be objective
 - Should be relatively easy and inexpensive to obtain and verify
 - Avoid from manipulating exposure information
 - Example, for auto insurance, annual miles vs. the number of car-year
 - Example, for product liability, number of product in use vs. number of product sold vs. annual sales
- Historical Precedence
 - Better exposure base may be discovered over time
 - Any change in an exposure base should be carefully considered
 - May lead a large premium swings for individual insureds
 - Will require a change in the rating algorithm, and thus rating systems
 - Historical data may need to be adjusted for analyses

Chapter 4 Exposures – Criteria for Exposure Bases

■ Exposure vs. Rating Variable

- Both relate to loss potential
- Both could be used in the rating structure
- Only one exposure base, but could have many rating variables in the rating structure
- Exposure base has linear and continuous relationship to expected losses; rating variable could be not

■ Auto Liability

- ☞ Territory R.V.
- ☞ Age R.V.
- ☞ Gender R.V.
- ☞ Limits R.V.
- ☞ # of cars E.B.

■ Fire Insurance

- ☞ Territory R.V.
- ☞ Use R.V.
- ☞ Construction type R.V.
- ☞ Fire protection R.V.
- ☞ Value of house R.V.
- ☞ # of house E.B.

Chapter 4 Exposures – Aggregation

[Example 4-1] Assuming the effective date = written date

Policy	Effective Date	Expiration Date	Exposure
A	1/1/2010	12/31/2010	1
B	4/1/2010	3/31/2011	1
C	7/1/2010	6/30/2011	1
D	10/1/2010	9/30/2011	1
E	1/1/2011	12/31/2011	1
F	4/1/2011	3/31/2012	1
G	7/1/2011	6/30/2012	1
H	10/1/2011	9/30/2012	1
I	1/1/2012	12/31/2012	1

Chapter 4 Exposures – Aggregation

■ CY

– Written Exposure

- The exposures arising from policies issued during the marked area

[Example 4-1] Written Exposure for CY 2010= Policies A, B, C, D = 4

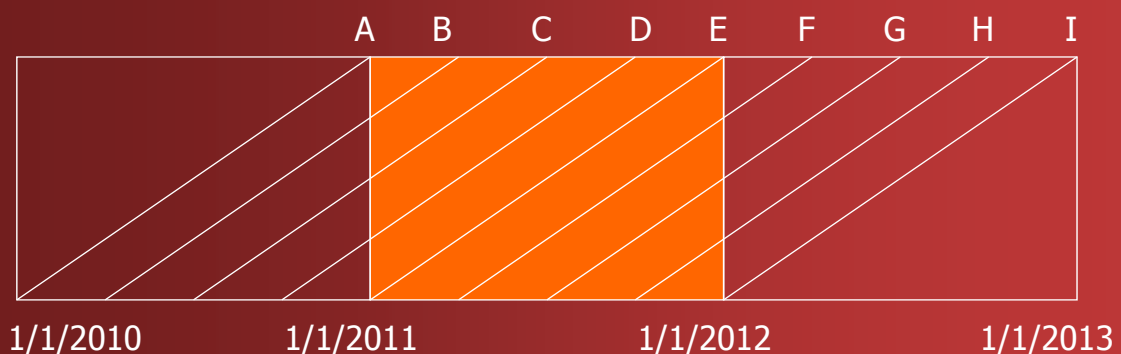
Written Exposure for CY 2011= Policies E, F, G, H = 4

- Exposures may be negative if there is any midterm adjustment, e.g. cancel

[Example 4-1] If policy C is cancelled on 4/1/2011

Written Exposure for CY 2010= Policies A, B, C, D = 4

Written Exposure for CY 2011= Policies C, E, F, G, H = $-0.25+4=3.75$



Chapter 4 Exposures – Aggregation

– Unearned Exposure

- The portion of the written exposures for which coverage has not been provided as of a certain point in time

- Uniform assumption is usually made for most line of businesses

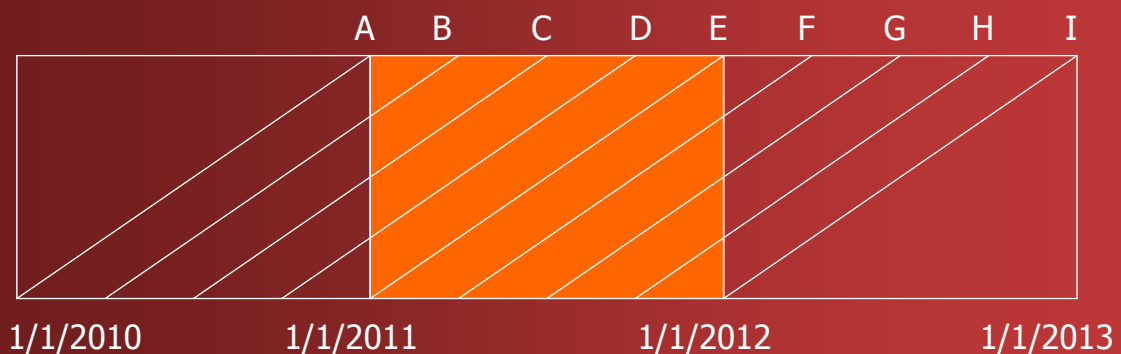
- The probability of a claim is evenly distributed during the whole policy term

[Example 4-1] Unearned Exposure at 12/31/2011= Policies F, G, H

$$= 1 \times 0.25 + 1 \times 0.5 + 1 \times 0.75 = 1.5$$

Unearned Exposure at 12/31/2010 = Policies B, C, D

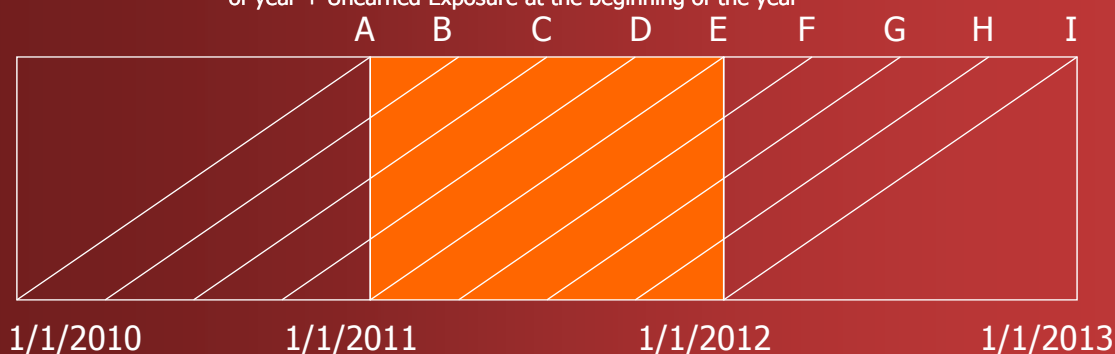
$$+ 1 \times 0.25 + 1 \times 0.5 + 1 \times 0.75 = 1.5$$



Chapter 4 Exposures – Criteria for Exposure Bases

– Earned Exposure

- The portion of the written exposures for which coverage has already been provided during the marked area
- Uniform assumption is usually made for most line of businesses
 - The probability of a claim is evenly distributed during the whole policy term
- Method 1
 - Directly determine the portion that has been earned
- Method 2
 - For a single policy, Earned Exposure = Written Exposure – Unearned Exposure
 - For a book of business Earned Exposure = Written Exposure – Unearned Exposure at the end of year + Unearned Exposure at the beginning of the year

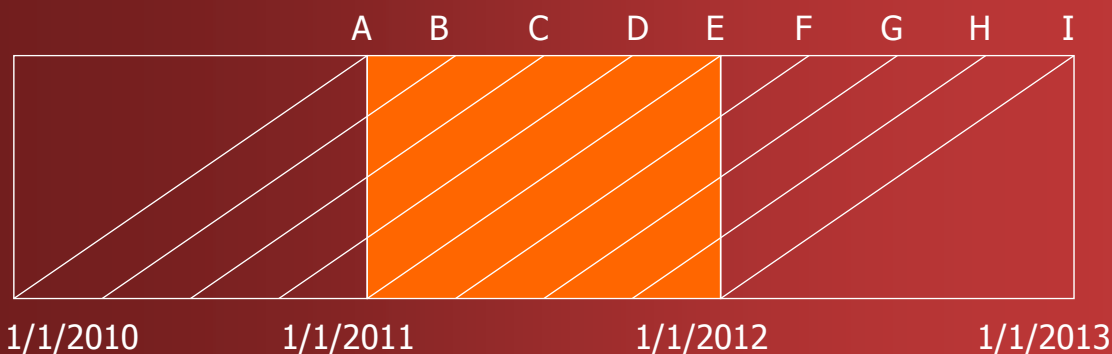


Chapter 4 Exposures – Aggregation

[Example 4-1]

Method 1 : Earned Exposure for CY 2011 = Policies B ~ H
 $= 1 \times 0.25 + 1 \times 0.5 + 1 \times 0.75 + 1 \times 1$
 $+ 1 \times 0.75 + 1 \times 0.5 + 1 \times 0.25$
 $= 4$

Method 2 : Earned Exposure for CY 2011
 $= 4 + 1.5 - 1.5$
 $= 4$

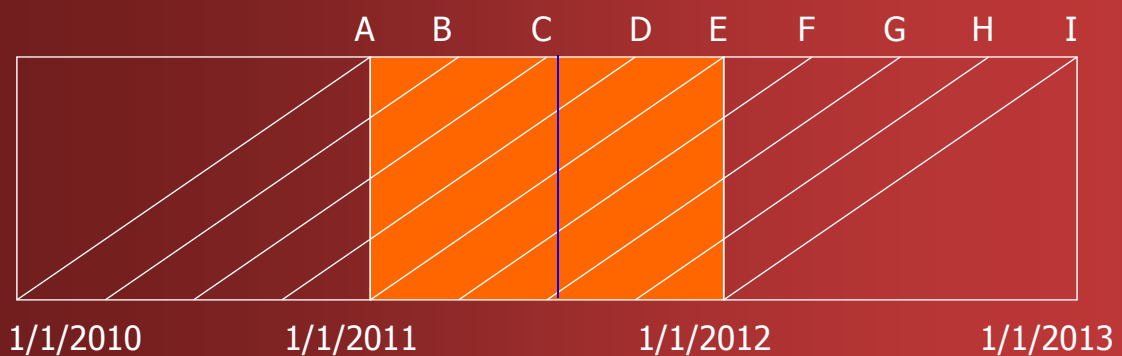


Chapter 4 Exposures – Aggregation

– In-force Exposure

- The number of exposures that are exposed to having a claim at a given point in time
- The exposures should be annualized

[Example 4-1] In-force Exposure at 7/1/2011= Policies D, E, F, G = 4



Chapter 4 Exposures – Aggregation

■ PY

– Written Exposure

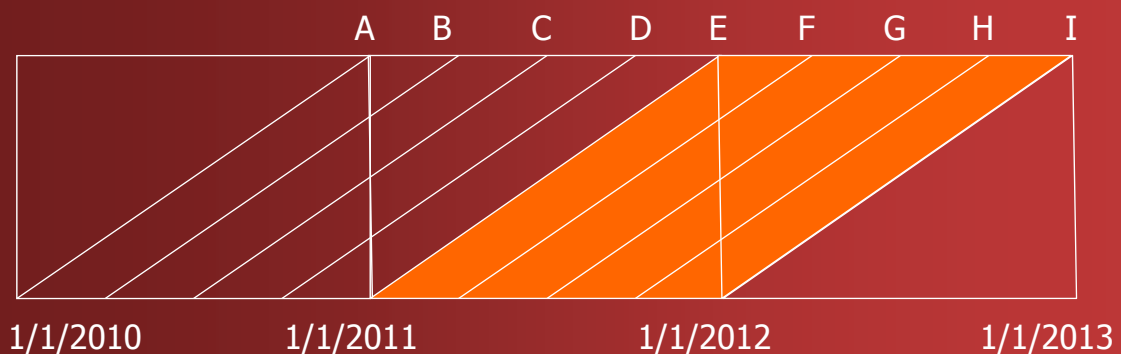
- The exposures with effective dates during the year

[Example 4-1] Written Exposure for PY 2011 at 12/31/2011= Policies E, F, G, H = 4

- Exposures may be changed over time

[Example 4-1] If policy G is cancelled on 4/1/2012

Written Exposure for PY 2011 at 12/31/2012 = 4 – 0.25 = 3.75



Chapter 4 Exposures – Aggregation

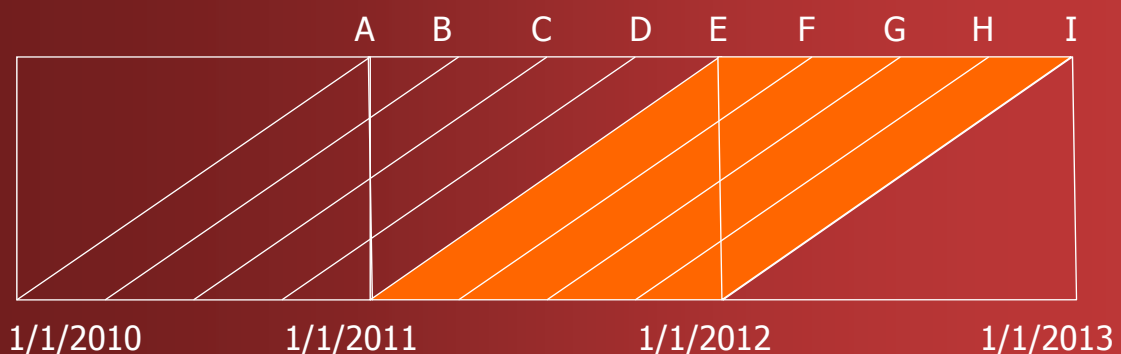
– Unearned Exposure

- The portion of the written exposures for which coverage has not been provided as of a certain point in time
- Uniform assumption is usually made for most line of businesses

[Example 4-1] Unearned Exposure for PY 2011 at 12/31/2011

$$= \text{Policies F, G, H} = 1 \times 0.25 + 1 \times 0.5 + 1 \times 0.75 = 1.5$$

$$\text{Unearned Exposure for PY 2011 at 12/31/2012} = 0$$



Chapter 4 Exposures – Criteria for Exposure Bases

– Earned Exposure

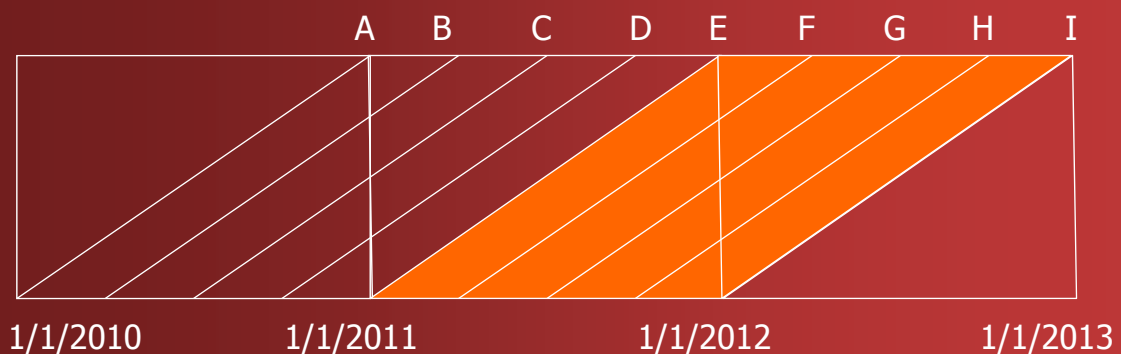
- The portion of the written exposures for which coverage has already been provided
- Uniform assumption is usually made for most line of businesses
- For any given PY, the earned exposure will be equal to written exposure after 24 months if the policy term is 1 year

■ Method 1

- Directly determine the portion that has been earned

■ Method 2

- $\text{Earned Exposure} = \text{Written Exposure} - \text{Unearned Exposure}$

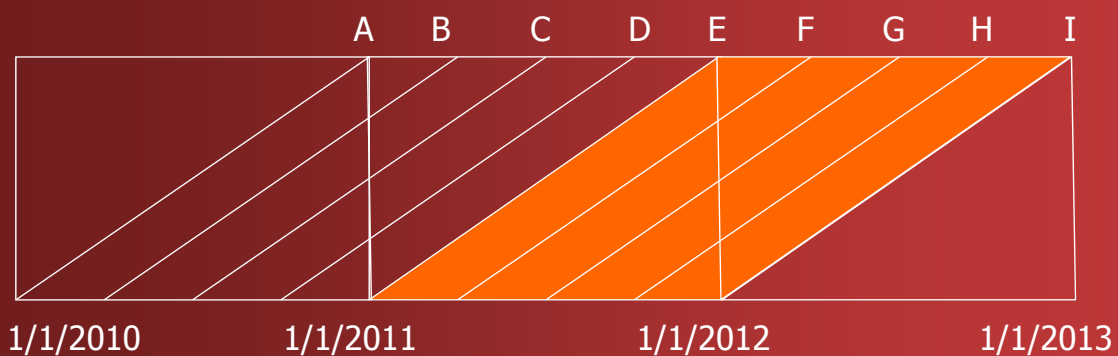


Chapter 4 Exposures – Aggregation

[Example 4-1]

Method 1 : Earned Exposure for PY 2011 at 12/31/2011 = Policies E ~ H
 $= 1 \times 1 + 1 \times 0.75 + 1 \times 0.5 + 1 \times 0.25 = 2.5$
 Earned Exposure for PY 2011 at 12/31/2012 = Policies E ~ H
 $= 4$

Method 2 : Earned Exposure for PY 2011 at 12/31/2011
 $= 4 - 1.5 = 2.5$
 Earned Exposure for PY 2011 at 12/31/2012
 $= 4 - 0$



Chapter 4 Exposures – Aggregation

[Sol] Summary of Example 4-1

Policy	Effective Date	Expiration Date	Exposure
A	1/1/2010	12/31/2010	1
B	4/1/2010	3/31/2011	1
C	7/1/2010	6/30/2011	1
D	10/1/2010	9/30/2011	1
E	1/1/2011	12/31/2011	1
F	4/1/2011	3/31/2012	1
G	7/1/2011	6/30/2012	1
H	10/1/2011	9/30/2012	1
I	1/1/2012	12/31/2012	1

Chapter 4 Exposures – Calculation of Blocks of Exposures

- Due to data availability, insurers may not determine the earned exposures by each policy
- Instead, an approximation may be used
- 1/2 method
 - Assuming all policies are 1-year
 - Auto, Homeowner, and PA insurances may apply
 - Engineering, Cargo, and Consumer Loan insurances may not apply
 - Uniform Assumptions
 - The probability of a claim is evenly distributed during the whole policy term
 - Engineering and Consumer Loan insurances may not be true
 - Policies are uniformly written during the year
 - Equivalent to the assumption that all policies are written on 7/1 of the year
 - For the insurances with large businesses such as Auto may be true
 - $\text{Earned Exposure} = \text{Written Exposure last year} \times \frac{1}{2} + \text{Written Exposure this year} \times \frac{1}{2}$

Chapter 4 Exposures – Calculation of Blocks of Exposures

- 1/8 method
 - All assumptions used in 1/2 method applies
 - Policies are uniformly written during each quarter
 - Equivalent to the assumption that all policies are written at mid-date of each quarter
 - Earned Exposure is determined as below



Chapter 4 Exposures – Calculation of Blocks of Exposures

■ 1/24 method

- All assumptions used in 1/2 method applies
 - Policies are uniformly written during each month
 - Equivalent to the assumption that all policies are written at 15th of each month
- The earned exposure percentages will be

Written Month	Assumed Effective Date	Earned % for CY 2011	Written Month	Assumed Effective Date	Earned % for CY 2011
Jan., 2010	1/15/2010	1/24	Jan, 2011	1/15/2011	23/24
Feb, 2010	2/15/2010	3/24	Feb, 2011	2/15/2011	21/24
Mar, 2010	3/15/2010	5/24	Mar, 2011	3/15/2011	19/24
Apr, 2010	4/15/2010	7/24	Apr, 2011	4/15/2011	17/24
May, 2010	5/15/2010	9/24	May, 2011	5/15/2011	15/24
Jun, 2010	6/15/2010	11/24	Jun, 2011	6/15/2011	13/24
Jul, 2010	7/15/2010	13/24	Jul, 2011	7/15/2011	11/24
Aug, 2010	8/15/2010	15/24	Aug, 2011	8/15/2011	9/24
Sep, 2010	9/15/2010	17/24	Sep, 2011	9/15/2011	7/24
Oct, 2010	10/15/2010	19/24	Oct, 2011	10/15/2011	5/24
Nov, 2010	11/15/2010	21/24	Nov, 2011	11/15/2011	3/24
Dec, 2010	12/15/2010	23/24	Dec, 2011	12/15/2011	1/24

Chapter 5 Premium



Chapter 5 Premium – Premium Aggregation

- All concepts for exposures will be applied to those for premium

Chapter 5 Premium – Adjustment to Premium

- For loss ratio method, both premium and losses are required
 - $LR = \text{Losses} / \text{Earned Premium}$
- In the ratemaking process, historical data is usually used
- Chapter 1 “Ratemaking is Prospective”
- Three basic adjustments
 - Bring the historical premium to the rate level currently in effect
 - Develop premium to ultimate levels if the premium is still changing
 - Project the historical premium to the premium level expected in the future

Chapter 5 Premium – Current Rate Level

- The rate levels during the experience period may not be the same
 - The current rate level may not be the same as historical rate levels
- The adequacy of current rate level is reviewed, not that of historical rate level
- The premium in the experience periods should be adjusted to the current rate level
- The adjusted premium is called on-level premium

Chapter 5 Premium – Current Rate Level

[Example 5-1]

- The written premium of a policy written in 2006 is \$1,000
- The rate has been increased by 10% in 2007
- What is the on-level premium at the end of 2008

[Sol]

The current rate should be 10% higher than the rate in 2006.

- ➡ The premium would be 10% higher.
- ➡ The on-level premium = $\$1,000 \times 1.1 = \$1,100$

Chapter 5 Premium – Current Rate Level

- Two methods
 - Extension of exposures method
 - Parallelogram method
- Extension of Exposures
 - Rerate each policy to restate the historical premium to the amount that would be charged under the current rates
 - Most accurate
 - Difficulties
 - Doing Manually is impossible
 - Rating software is needed
 - The rating structure could be changed
 - The policy detail information is required

Chapter 5 Premium – Current Rate Level

[Example 5-2] Please determine the on-level premium for the policy which was effective on 3/1/2011, and had 10 class Y exposures. Assuming

- Premium = Exposure x Base Rate x Class Relativity+ Fee
- The historical rates are shown below

Rate Level Group	Eff. Date	Overall Ave. Rate Change	Base Rate	Relativities			Policy Fee
				X	Y	Z	
1	Initial		900	1.00	0.60	1.10	1,000
2	7/1/2010	5.0%	950	1.00	0.60	1.10	1,000
3	1/1/2011	10.0%	1,045	1.00	0.60	1.10	1,100
4	4/1/2012	-1.0%	1,045	1.00	0.70	1.05	1,090

[Sol] On-Level Premium = $10 \times 1,045 \times 0.7 + 1,090$
= 8,405

Even though the actual premium charged
= $10 \times 1,045 \times 0.6 + 1,100 = 7,370$

Chapter 5 Premium – Current Rate Level

■ Parallelogram Method

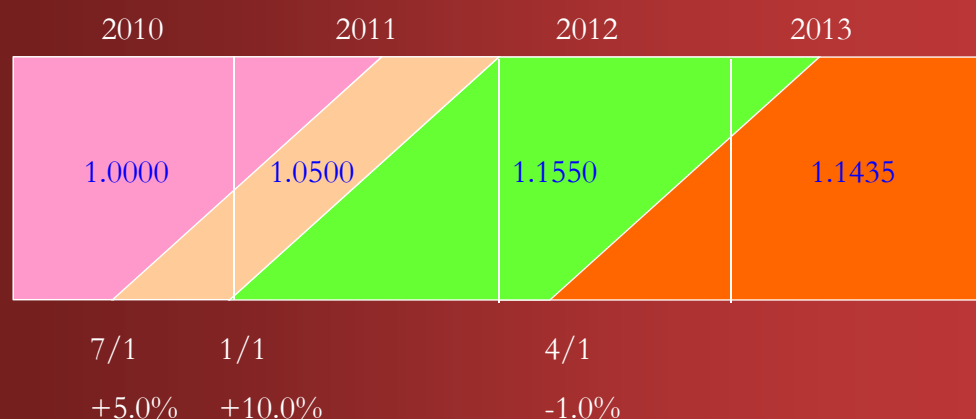
- Not most accurate, but used most often in practical
- Simple geometric relationship
- Uniform assumptions
 - The probability of a claim is evenly distributed during the whole policy term
 - That is, risk is uniformly disposed over policy period
 - Policies are uniformly written during the experience period
- Rate change history must be available
- Length of policy is important
 - 1 year, in general
 - If not 1 year, same approach can be applied

Chapter 5 Premium – Current Rate Level

[Example 5-3] Continue with the Example 5-2, please determine the on-level factor for the years 2011 and 2012

[Sol]

■ Rate Indexes



Chapter 5 Premium – Current Rate Level

■ Average Earned Rate

– CY 2011

$$\text{Ave ERate} = (\frac{1}{2} \times 0.5 \times 0.5) \times 1.0000 + (1 - \frac{1}{2} \times 0.5 \times 0.5 - \frac{1}{2} \times 1 \times 1) \times 1.0500 + (\frac{1}{2} \times 1 \times 1) \times 1.1550 = 1.0963$$

– CY 2012

$$\text{Ave ERate} = (1 - \frac{1}{2} \times 0.75 \times 0.75) \times 1.1550 + (\frac{1}{2} \times 0.75 \times 0.75) \times 1.1435 = 1.1518$$

■ On-Level Factor

– Current rate (at the end of year 2012) = 1.1435

– On-Level Factor = Current Rate / Ave ERate

$$F_{2011} = 1.1435 / 1.0963 = 1.0431$$

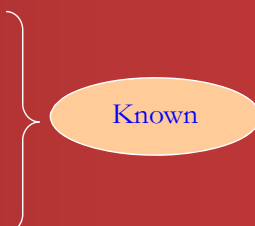
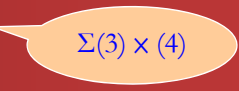
$$F_{2012} = 1.1435 / 1.1518 = 0.9928$$

Chapter 5 Premium – Current Rate Level

- If the rates have been moving upward, the on-level factor should be bigger than one and decreased
- If the rates have been moving downward, the on-level factors should be smaller than one and increased
- Historical rate change
 - Announced industrial rate change may be not appropriate for an individual company
 - Company's rate change should be determined using company's mix of business
 - In-Force premium should be used to determin the mix of business

Chapter 5 Premium – Current Rate Level

[Example 5-4]

(1)	(2)	(3)=(2)/(2)Total	(4)	
Use	In-Force Premium as of 7/1/2005	Mix of Business	Rate Change	
A	1,000	14.3%	12.5%	
B	2,000	28.6%	15.2%	
C	1,500	21.4%	6.5%	
D	2,500	35.7%	-8.0%	
Total	7,000		4.7%	

Chapter 5 Premium – Premium Development

- The ultimate amount of premium for the experience period may be unknown at the time of the analysis
 - Incomplete year of data is used
 - if policy year is used, the adjustment to ultimate is needed
 - Premium audits are used for the line of business
 - The actual premium is determined when the actual exposure is known at the end of the policy term, e.g. product liability, workers compensation
 - most of line of business do not utilize the premium audits
- Analyze historical patterns of premium to estimate the development factor

Chapter 5 Premium — Premium Development

■ Premium Audits

- The actual premium is determined when the actual exposure is known at the end of the policy term
- E.g. product liability, workers compensation

Chapter 5 Premium — Premium Development

[Example 5-5]

- A workers compensation carrier writes one policy per month in 2011
- Estimated premium for each policy is booked at policy inception for \$500,000
- Premium on every policy develops upward by 8% at the first audit, six months after the policy expires

[Sol]

Premium for PY 2011 :

- On 12/31/2012 : $6 \times 500,000 \times 1.08 + 6 \times 500,000 = 6,240,000$
- On 12/31/2013 : $12 \times 500,000 \times 1.08 = 6,480,000$

Premium development factor from 24 months to 36 months
(after the start of the policy) = $6,480,000 / 6,240,000 = 1.0385$

Chapter 5 Premium — Premium Trend

- Average premium level can change over time due to changes in the characteristics of the policies written
 - Rating characteristic
 - Gender: increase in the number of male policyholders may increase the premium for auto
 - Value of home: as increase of the value of home, the average premium for homeowner policy may increase
 - Deductible change
- It is important to adjust the historical premium to the level expected during the future time period
- Need to distinguish one-time event or a shift that is expected to continue in the future
- Need to judgmentally incorporate any additional shifts that are reasonably expected to happen in the future

Chapter 5 Premium — Premium Trend

- Approaches to measure the premium trend
 - Examine how premium distribution by individual rating variable have shifted over time
 - Deductible shift
 - Model Year shift
 - Examine all premium shifts simultaneously
 - Average premium per exposure, not per policy
 - Both earned premium and written premium can be used
 - Written premium is a leading indicator of trends
 - Premium should be on-leveled
 - If not, the impact of rate change will be double counted
 - Quarterly data is better if available
 - Any reasonable statistics approach can be used
 - Linear model
 - Exponential model
 - Any others

Chapter 5 Premium — Premium Trend

[Example 5-6]

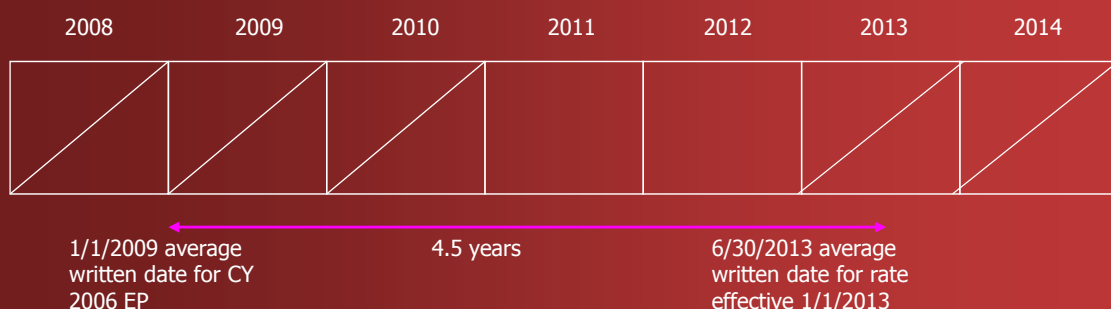
(1)	(2)	(3)	$\frac{(2)}{(3)}$ (4)	$\frac{(4)}{\text{Prior Year}(4)}$ (5)
Quarter	WP @ Current Rate Level	Written Exposures	Average WP @ Current Rate Level	Annual Change
2009Q1	323,189	453	713	
2009Q2	328,325	458	717	
2009Q3	333,502	463	720	
2009Q4	338,722	468	724	
2010Q1	343,667	472	728	2.1%
2010Q2	348,696	477	731	2.0%
2010Q3	353,027	481	734	1.9%
2010Q4	358,099	485	738	2.0%
2011Q1	361,755	488	741	1.8%
2011Q2	367,654	493	746	2.0%
2011Q3	372,305	497	749	2.1%
2011Q4	377,253	501	753	2.0%

Chapter 5 Premium – Premium Trend

■ One-Step Trending

- Trend factor = $(1 + \text{Trend})^{\text{trend period}}$
- Trend period: the average written date in experience period to average written date in the future
- Trended Premium = Premium $\times$ Trend Factor

[Example 5-7] What is the trend period for CY 2009 EP, if the new rate will be effective 1/1/2013 for one year?



Chapter 5 Premium – Premium Trend

■ Two-Step Trending

– Will be used if actuary believes the historical premium trend is different from the future trend

– Step 1

■ Trend from the average written date in the historical period to the average written date of the latest period

$$\text{Current Trend Factor} = \frac{\text{Latest Average WP at Current Rate Level}}{\text{Historical Average EP at Current Rate Level}}$$

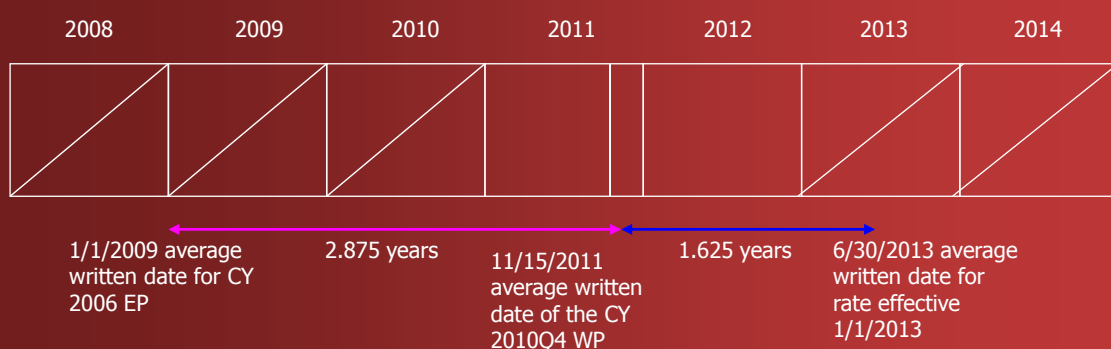
Chapter 5 Premium – Premium Trend

– Step 2

■ Projected Trend factor = $(1 + \text{Trend})^{\text{trend period}}$

■ Trend Period: from the average written date of the latest period to the average written date in the future

[Example 5-8] Following the example 5-7



Chapter 5 Premium – Premium Trend

– Total Premium Trend Factor=
Current Trend Factor × Projected Trend Factor

[Example 5-9]

Please determine the trended CY 2009 EP at current rate level using the information given below if the new rate will be effective 1/1/2013

(1) CY 2009 EP at Current Rate Level	1,440,788
(2) CY 2009 Earned Exposure	1,947
(3) 2011 Q4 Ave. WP at Current Rate Level	753
(4) Selected future trend	2.0%

Chapter 5 Premium – Premium Trend

[Sol]

(1)/(2)=	(5) CY 2009 Ave. EP at Current Rate Level	740
(3)/(5)=	(6) Current Trend Factor	1.0176
[Note1.]	(7) Trend Period	1.6250
(1+(4))^(7)=	(8) Projected Trend Factor	1.0327
(6)×(8)=	(9) Total Premium Trend Factor	1.0508
(1)×(9)=	(10) Trended CY 2009 EP at Current Rate Level	1,514,036

[Note]

1. Trend period is from 11/15/2011 to 6/30/2013

Chapter 6

Losses and LAE



Chapter 6 Losses and LAE — Aggregation

- C.Y.
 - All loss transactions occur during the 12-month calendar year
 - Incurred (Reported) Losses
 - = Losses paid during the year
 - + Loss reserves at the end of the year
 - Loss reserves at the beginning of the year
- [Note] Depending on the purpose, loss reserves may or may not include IBNR
- Incurred losses are fixed at the end of C.Y.

Chapter 6 Losses and LAE — Aggregation

■ A.Y.

- All loss transactions for claims that occur during the year being evaluated
- Incurred (Reported) Losses
= Losses paid + loss reserves as of the valuation date for all claims due to accident occurred in the 12 months period

[Note] Depending on the purpose, loss reserves may or may not include IBNR

- Incurred losses may change over time after the end of the year

Chapter 6 Losses and LAE — Aggregation

■ P.Y.

- All loss transactions on policies that were effective during the year being evaluated
- Incurred (Reported) Losses
= Losses paid + loss reserves as of the valuation date for all claims arising from policies effective during the 12 months period

[Note] Depending on the purpose, loss reserves may or may not include IBNR

- Incurred losses may change over time after the end of the year

Chapter 6 Losses and LAE — Aggregation

■ R.Y.

- All loss transactions for claims that are reported during the year being evaluated
- Incurred (Reported) Losses
= Losses paid + loss reserves as of the valuation date for all claims due to accident occurred in the 12 months period

[Note] In this aggregation, there is not pure IBNR

- Incurred losses may change over time after the end of the year

Chapter 6 Losses and LAE — Aggregation

[Example 6-1]

- Please determine the loss ratio for CY, AY, and PY 2009, 2010, and 2011 using the following data below

Policy #	Policy Effective Date	Written Premium	Date of Loss	Report Date	Transaction Date	Incremental Payment	Case Reserve
A001	2009/7/1	18,000	2009/11/1	2009/11/19	2009/11/19	0	10,000
					2010/2/1	1,000	9,000
					2010/9/1	7,000	2,500
					2011/1/15	3,000	0
A002	2009/9/1	16,800	2010/2/14	2010/2/14	2010/2/14	5,000	10,000
					2010/11/1	8,000	4,000
					2011/3/1	1,000	0

Chapter 6 Losses and LAE — Aggregation

[Sol] P.Y.

@2009/12/31					
PY	EP	Paid Losses	Case Reserve_End	Reported Losses	Loss Ratio
2009	14,600	0	10,000	10,000	68.49%
@2010/12/31					
PY	EP	Paid Losses	Case Reserve_End	Reported Losses	Loss Ratio
2009	34,800	21,000	6,500	27,500	79.02%
2010	0	0	0	0	#DIV/0!
@2011/12/31					
PY	EP	Paid Losses	Case Reserve_End	Reported Losses	Loss Ratio
2009	34,800	25,000	0	25,000	71.84%
2010	0	0	0	0	#DIV/0!
2011	0	0	0	0	#DIV/0!

Chapter 6 Losses and LAE — Aggregation

A.Y.

@2009/12/31					
AY	EP	Paid Losses	Case Reserve_End	Reported Losses	Loss Ratio
2009	14,600	0	10,000	10,000	68.49%
@2010/12/31					
AY	EP	Paid Losses	Case Reserve_End	Reported Losses	Loss Ratio
2009	14,600	8,000	2,500	10,500	71.92%
2010	20,200	13,000	4,000	17,000	84.16%
@2011/12/31					
AY	EP	Paid Losses	Case Reserve_End	Reported Losses	Loss Ratio
2009	14,600	11,000	0	11,000	75.34%
2010	20,200	14,000	0	14,000	69.31%
2011	0	0	0	0	#DIV/0!

Chapter 6 Losses and LAE — Aggregation

C.Y.

CY	Policy A001				Policy A002			
	EP	Paid Losses	Change in Case	Reported Losses	EP	Paid Losses	Change in Case	Reported Losses
2009	9,000	0	10,000	10,000	5,600	0	0	0
2010	9,000	8,000	-7,500	500	11,200	13,000	4,000	17,000
2011	0	3,000	-2,500	500	0	1,000	-4,000	-3,000
Total	18,000	11,000		11,000	16,800	14,000		14,000

CY	Total				
	EP	Paid Losses	Change in Case	Reported Losses	Loss Ratio
2009	14,600	0	10,000	10,000	68.49%
2010	20,200	21,000	-3,500	17,500	86.63%
2011	0	4,000	-6,500	-2,500	#DIV/0!
Total	34,800	25,000		25,000	71.84%

Chapter 6 Losses and LAE — Adjustments to Losses

- Loss adjustments includes
 - Remove extraordinary events
 - Restate losses to the benefit and cost levels expected in the future
 - Loss trend
 - Develop immature losses to ultimate

Chapter 6 Losses and LAE — Extraordinary Losses

- Extraordinary Losses include
 - Large individual losses
 - Catastrophe losses
- Large individual losses
 - If actual shock losses are included, indicated rates may increase immediately after a year with shock losses
 - May exclude
 - The portion above some predetermined threshold
 - The entire losses if the losses are excess of the threshold
 - A provision will be added to reflect the expected shock losses based on a long-term view
 - Indicated rates will be underestimated if not do so

Chapter 6 Losses and LAE — Extraordinary Losses

- Selection of the threshold
 - Basic limit is one of the choice
 - The rate level indication is the rate for the basic limit
 - The premium used must be adjusted to the basic limit
 - ILF will then be determined for the limits other than the basic limit
 - A point at which the losses are extraordinary and their inclusion causes volatility in the rates
 - Including as many losses as possible and minimizing the volatility
 - Examining the size of distribution and set the threshold at a given percentile, such as the 99th percentile
- When capping the historical losses, it is better to use the trended reported losses

Chapter 6 Losses and LAE — Aggregation

[Example 6-2]

Threshold : 1,000,000

	(1)	(2)	(3)	$= (3) - 1,000,000 \times (2)$ (4)	$= (1) - (4)$ (5)	$= (4) / (5)$ (6)
AY	Reported Losses	Number of Excess Claims	Ground-up Excess Losses	Losses Excess of Threshold	Non-Excess Losses	Excess Ratio
1996	118,369,707	5	6,232,939	1,232,939	117,136,768	1.1%
1997	117,938,146	1	1,300,000	300,000	117,638,146	0.3%
1998	119,887,865	3	3,923,023	923,023	118,964,842	0.8%
1999	118,488,983	0	0	0	118,488,983	0.0%
2000	122,329,298	7	12,938,382	5,938,382	116,390,916	5.1%
2001	120,157,205	3	3,824,311	824,311	119,332,894	0.7%
2002	123,633,881	0	0	0	123,633,881	0.0%
2003	124,854,827	1	3,000,000	2,000,000	122,854,827	1.6%
2004	125,492,840	0	0	0	125,492,840	0.0%
2005	127,430,355	6	13,466,986	7,466,986	119,963,369	6.2%
2006	123,245,269	3	4,642,423	1,642,423	121,602,846	1.4%
2007	123,466,498	0	0	0	123,466,498	0.0%
2008	129,241,078	10	17,038,332	7,038,332	122,202,746	5.8%
2009	123,302,570	0	0	0	123,302,570	0.0%
2010	123,408,837	3	4,351,805	1,351,805	122,057,032	1.1%
Total	1,841,247,359	42	70,718,201	28,718,201	1,812,529,158	1.6%
		1+(6)Total =	(7)	Excess Loss Factor		1.016

Chapter 6 Losses and LAE — Changes in Coverage or Benefit Levels

- Coverage or benefit levels may change due to
 - Changes initiated by insurers
 - Changes in law
- Any changes in coverage or benefit levels should be reflected in the historical loss data
 - May have both direct and indirect impacts
 - Difficult to measure the indirect impacts
- Need to understand how the changes impact the losses
 - Losses arising from the policies effective after a certain date
 - Losses arising from the accidents occurred after a certain date
 - Loss payments made after a certain date
- Two approaches
 - Restate each historical claim to future benefit levels
 - Most accurate, but cumbersome
 - Measure the average effect

Chapter 6 Losses and LAE — Changes in Coverage or Benefit Levels

[Example 6-3]

Current:	5,000		
Proposed:	3,000		
		$= \text{Min}((1), 3000)$	$= (2)/(1) - 1$
	(1)	(2)	(3)
	Losses Capped at		
Claim #	Current Level	Proposed Level	Effect of Change
1	1,100	1,100	0.0%
2	2,350	2,350	0.0%
3	3,700	3,000	-18.9%
4	4,100	3,000	-26.8%
5	5,000	3,000	-40.0%
6	5,000	3,000	-40.0%
Total	21,250	15,450	-27.3%

Chapter 6 Losses and LAE — Changes in Coverage or Benefit Levels

[Example 6-4]

- The compensation rate is 66.7% of the worker's pre-injury wage
- The state average weekly wage (SAWW) is currently \$1,000
- The minimum indemnity benefit remains at 50% of the SAWW
- The maximum benefit is decreasing from 100% of the SAWW to 83.3% of the SAWW
- The distribution of workers according to how their wages compare to the SAWW is as follows:

Ratio to SWAA	# of Workers	Total Weekly Wages
< 50%	7	3,000
50% - 75%	24	16,252
75% - 100%	27	23,950
100% - 125%	19	23,048
125% - 150%	12	16,500
150% >	11	17,250
Total	100	100,000

Chapter 6 Losses and LAE — Changes in Coverage or Benefit Levels

[Sol]

- Workers with wages less than 75.0% of SAWW will be compensated at the minimum level
 - 75.0% = 50.0% / 66.7%
- Workers with wages more than 150.0% of SAWW will be compensated at the current maximum level
 - 150.0% = 100.0% / 66.7%
- Workers with wages more than 125.0% of SAWW will be compensated at the proposed maximum level
 - 125.0% = 83.3% / 66.7%
- The current benefits and proposed benefits are shown below

Chapter 6 Losses and LAE — Changes in Coverage or Benefit Levels

(1)	(2)	(3)	(4)	(5)
Ratio to SWAA	# of Workers	Total Weekly Wages	Current Benefits	Proposed Benefits
< 50%	7	3,000	3,500	3,500
50% - 75%	24	16,252	12,000	12,000
75% - 100%	27	23,950	15,967	15,967
100% - 125%	19	23,048	15,365	15,365
125% - 150%	12	16,500	11,000	10,000
150% >	11	17,250	11,000	9,167
Total	100	100,000	68,832	65,999

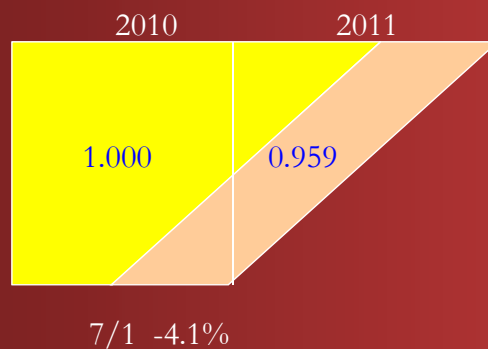
(6) Benefit Change Effect -4.1%=(5)Total / (4) Total-1

[Note]

1. (4)= (2)×1000×50% For (1)<75%
(3)×66.7% For Others
2. (5)= (2)×1000×100% For (1)>150%
(2)×1000×50% For (1)<75%
(3)×66.7% For Others
(2)×1000×83.3% For (1)>125%

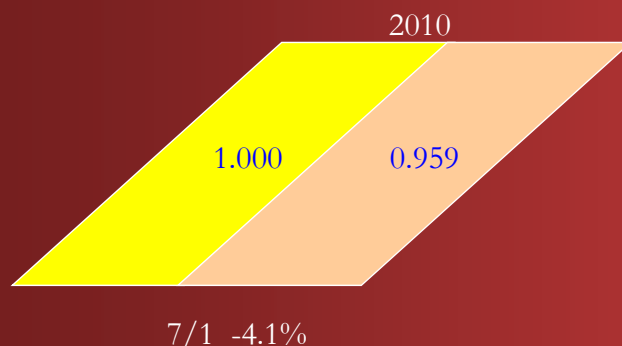
Chapter 6 Losses and LAE — Changes in Coverage or Benefit Levels

- If the change is on policies effective on and after 7/1/2010
 - For AY loss data
 - Proposed benefit level: 0.959
 - Average Historical benefit level: $1.000 \times (1 - 0.125) + 0.959 \times 0.125 = 0.995$
 - Adjustment Factor = Proposed Benefit Level / Average Historical Benefit Level = 0.964



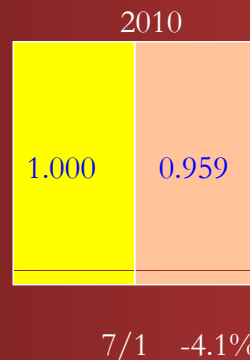
Chapter 6 Losses and LAE — Changes in Coverage or Benefit Levels

- For PY loss data
 - Proposed benefit level: 0.959
 - Average Historical benefit level: $1.000 \times (1 - 0.5) + 0.959 \times 0.5 = 0.979$
 - Adjustment Factor = Proposed Benefit Level / Average Historical Benefit Level = 0.979



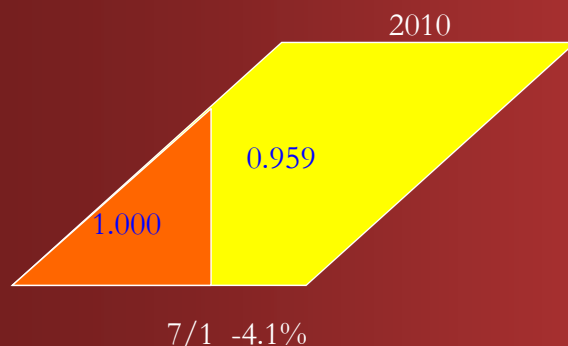
Chapter 6 Losses and LAE — Changes in Coverage or Benefit Levels

- If the change is on losses arising from accidents on and after 7/1/2010
 - For AY loss data
 - Proposed benefit level: 0.959
 - Average Historical benefit level: $1.000 \times (1 - 0.5) + 0.959 \times 0.5 = 0.979$
 - Adjustment Factor = Proposed Benefit Level / Average Historical Benefit Level = 0.979



Chapter 6 Losses and LAE — Changes in Coverage or Benefit Levels

- For PY loss data
 - Proposed benefit level: 0.959
 - Average Historical benefit level: $1.000 \times 0.125 + 0.959 \times (1 - 0.125) = 0.964$
 - Adjustment Factor = Proposed Benefit Level / Average Historical Benefit Level = 0.995



Chapter 6 Losses and LAE — Loss Development

- Why need to develop the losses to ultimate
 - The existing loss data may not completely develop
 - Some losses haven't been reported
 - If not project to ultimate, the rate indication will be underestimated
- Methods to project the ultimate losses
 - Loss Development Method (Chain Ladder Method, Link Ratio Method)
 - Bornhuetter-Ferguson Method (BF Method)

Chapter 6 Losses and LAE — Loss Development

- Loss Development Method
 - Assumption
 - Claims move from unreported to reported-and- unpaid to paid in a pattern that is sufficiently consistent
 - Loss Triangle
 - Data are arrayed by AY and development age
 - Development Age = Valuation Date – Accident Date

Chapter 6 Losses and LAE — Loss Development

- Loss Development Factor (LDF)
 - Age-to-Age LDF (a.k.a Link Ratio, Age LDF, or Age Factor, or $LDF_{x,x+1}$, hereafter)
 - The relationship between two consecutive valuations
 - The development of losses from one valuation to the followed valuation
 - $LDF_{x,x+1} = \text{Loss at } x+1 / \text{Loss at } x$
 - [Note] $LDF_{ult,ult+1} = 1$
 - Age-to-Ultimate LDF (Ultimate LDF, or LDF, or LDF_x , hereafter)
 - The development of losses from one valuation to ultimate
 - $LDF_x = \prod_{t=x}^{ult-1} LDF_{t,t+1}$
 - 1/LDF represents the % of ultimate that has been developed
- Ultimate Loss
 - Ultimate Loss for valuation of x = Loss at x $\times LDF_x$

Chapter 6 Losses and LAE — Loss Development

[Example 6-5]

- Please determine the ultimate losses using the data below

Case-Incurred Losses and ALAE

Unit: '000

Accident Year	As of 12/31					
	2003	2004	2005	2006	2007	2008
2003	2,116	3,129	3,543	3,707	3,854	3,929
2004		2,316	3,527	3,993	4,182	4,339
2005			2,744	4,052	4,593	4,797
2006				3,130	4,589	5,230
2007					3,625	5,381
2008						3,920

Chapter 6 Losses and LAE — Loss Development

[Sol]

■ Step 1: Transfer to Loss Triangle

Case-Incurred Losses and ALAE Triangle

Unit: '000

Accident Year	As of 12/31					
	12	24	36	48	60	72
2003	2,116	3,129	3,543	3,707	3,854	3,929
2004	2,316	3,527	3,993	4,182	4,339	
2005	2,744	4,052	4,593	4,797		
2006	3,130	4,589	5,230			
2007	3,625	5,381				
2008	3,920					

Chapter 6 Losses and LAE — Loss Development

■ Step 2: Determine the Age-to-Age LDF and Age-to-Ultimate LDF

Age LDF & Ultimate LDF

Accident Year	As of 12/31					
	12-24	24-36	36-48	48-60	60-72	72-Ult
2003	1.4785	1.1326	1.0463	1.0396	1.0194	1.0000
2004	1.5230	1.1320	1.0474	1.0375		
2005	1.4768	1.1336	1.0444			
2006	1.4661	1.1397				
2007	1.4841					
Selected	1.4800	1.1350	1.0450	1.0385	1.0200	1.0000
Ult LDF	1.8594	1.2564	1.1069	1.0593	1.0200	1.0000

Chapter 6 Losses and LAE — Loss Development

■ Step 3: Determine the Ultimate Losses

Unit: '000				
AY	Age	Loss and ALAE	Ult LDF	Ultimate
2003	72	3,929	1.0000	3,929
2004	60	4,339	1.0200	4,426
2005	48	4,797	1.0593	5,082
2006	36	5,230	1.1069	5,790
2007	24	5,381	1.2564	6,760
2008	12	3,920	1.8594	7,288
Total		27,595		33,274

Chapter 6 Losses and LAE — Loss Trend

■ Loss Trend

- The loss will change due to inflation, jury award, legal environment, and etc.
- Including frequency and severity trend
 - Most often do the two trends separately
 - The combine the two trends to determine the pure premium trend
 - Frequency Trend
 - More strict traffic law may reduce the bodily injury (BI) frequency
 - Court decision may increase the liability frequency
 - Severity Trend
 - Inflation is a nature cause of the severity
 - Court decision may increase the severity of the liability
 - Improvement of the medical equipment may increase the severity of BI

Chapter 6 Losses and LAE — Loss Trend

- Data used
 - For more stable, short-tailed lines of business, CY paid losses are better
 - For more volatile, long-tailed lines of business, AY reported losses are better
 - Using 12-month rolling data can smooth out the effect of seasonality
 - Catastrophe losses are normally excluded
- May consider any external information, such as price-index
- The trend in the experience period and future may be different
 - Based on actuaries' judgment
 - Use two steps to reflect the different trends

Chapter 6 Losses and LAE — Loss Trend

[Example 6-6]

- Please determine the trends for frequency, severity, and pure premium

CY (Year Ending Quarter)	Earned Exposure	Closed Claim Count	Paid Losses
2009Q1	131,911	7,745	8,220,899
2009Q2	132,700	7,785	8,381,016
2009Q3	133,602	7,917	8,594,389
2009Q4	135,079	7,928	8,705,108
2010Q1	137,384	7,997	8,816,379
2010Q2	138,983	8,037	8,901,163
2010Q3	140,396	7,939	8,873,491
2010Q4	140,997	7,831	8,799,730
2011Q1	140,378	7,748	8,736,859
2011Q2	139,682	7,719	8,676,220
2011Q3	138,982	7,730	8,629,925
2011Q4	138,984	7,790	8,642,835
2012Q1	139,155	7,782	8,602,105
2012Q2	139,618	7,741	8,535,327
2012Q3	139,996	7,720	8,466,272
2012Q4	140,141	7,691	8,412,159
2013Q1	140,754	7,735	8,513,679
2013Q2	141,534	7,769	8,614,224
2013Q3	141,800	7,755	8,702,135
2013Q4	142,986	7,778	8,761,588

Chapter 6 Losses and LAE — Loss Trend

[Sol]

(1) CY (Year Ending Quarter)	(2) Earned Exposure	(3) Closed Claim Count	(4) Paid Losses	(5) Frequency	(6) Annual Change	(7) Severity	(8) Annual Change	(9) Pure Premium	(10) Annual Change
2009Q1	131,911	7,745	8,220,899	0.0587		1,061.45		62.32	
2009Q2	132,700	7,785	8,381,016	0.0587		1,076.56		63.16	
2009Q3	133,602	7,917	8,594,389	0.0593		1,085.56		64.33	
2009Q4	135,079	7,928	8,705,108	0.0587		1,098.02		64.44	
2010Q1	137,384	7,997	8,816,379	0.0582	-0.9%	1,102.46	3.9%	64.17	3.0%
2010Q2	138,983	8,037	8,901,163	0.0578	-1.4%	1,107.52	2.9%	64.04	1.4%
2010Q3	140,396	7,939	8,873,491	0.0565	-4.6%	1,117.71	3.0%	63.20	-1.7%
2010Q4	140,997	7,831	8,799,730	0.0555	-5.4%	1,123.70	2.3%	62.41	-3.2%
2011Q1	140,378	7,748	8,736,859	0.0552	-5.2%	1,127.63	2.3%	62.24	-3.0%
2011Q2	139,682	7,719	8,676,220	0.0553	-4.4%	1,124.01	1.5%	62.11	-3.0%
2011Q3	138,982	7,730	8,629,925	0.0556	-1.6%	1,116.42	-0.1%	62.09	-1.8%
2011Q4	138,984	7,790	8,642,835	0.0560	0.9%	1,109.48	-1.3%	62.19	-0.4%
2012Q1	139,155	7,782	8,602,105	0.0559	1.3%	1,105.38	-2.0%	61.82	-0.7%
2012Q2	139,618	7,741	8,535,327	0.0554	0.3%	1,102.61	-1.9%	61.13	-1.6%
2012Q3	139,996	7,720	8,466,272	0.0551	-0.9%	1,096.67	-1.8%	60.48	-2.6%
2012Q4	140,141	7,691	8,412,159	0.0549	-2.1%	1,093.77	-1.4%	60.03	-3.5%
2013Q1	140,754	7,735	8,513,679	0.0550	-1.7%	1,100.67	-0.4%	60.49	-2.2%
2013Q2	141,534	7,769	8,614,224	0.0549	-1.0%	1,108.79	0.6%	60.86	-0.4%
2013Q3	141,800	7,755	8,702,135	0.0547	-0.8%	1,122.13	2.3%	61.37	1.5%
2013Q4	142,986	7,778	8,761,588	0.0544	-0.9%	1,126.46	3.0%	61.28	2.1%

Chapter 6 Losses and LAE — Loss Trend

Number of Points	Exponential Fit		
	Frequency	Severity	Pure Premium
20	-1.7%	0.5%	-1.2%
16	-1.3%	-0.1%	-1.4%
12	-0.7%	-0.2%	-0.9%
8	-1.3%	1.2%	-0.1%
6	-0.9%	2.5%	1.6%
4	-1.4%	3.3%	1.9%

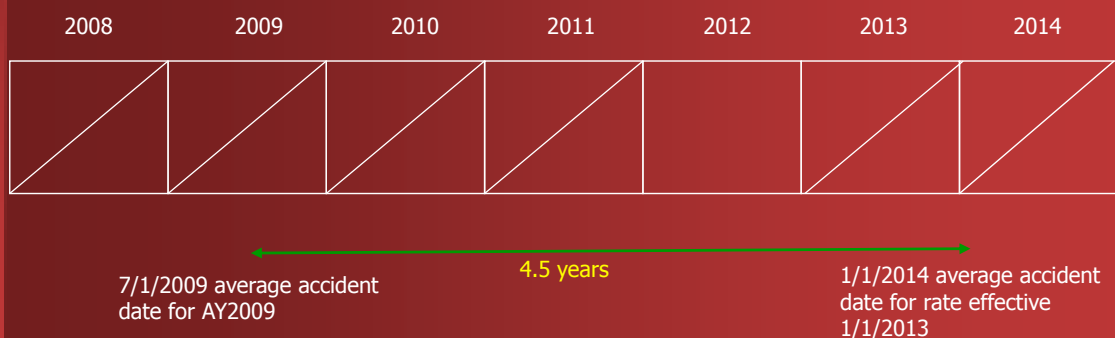
Chapter 6 Losses and LAE — Loss Trend

■ Trend Period

- Average accident date in experience period to average accident date in the future

[Note] The loss trend period is different from the premium trend period, but same length of the trend

[Example 6-7] What is the loss trend period for AY 2009 losses, if the new rate will be effective 1/1/2013 for one year?



Chapter 6 Losses and LAE — Loss Trend

■ Effects of Limits on Severity Trend

- The insured limit for liability insurance will have a impact on severity trend
- Higher layer, larger impact of trend
- Higher limit, higher trend
- Different effects among layers

X: unlimited loss size

L: lower bound of layer

U: upper bound of layer

T: severity increase rate

Original Loss Size	Rate of Increase in Layer
$X \leq L$	Undefined
$L < X \leq U/(1+T)$	$(X(1+T)-L)/(X-L)-1=XT/(X-L)$
$U/(1+T) < X \leq U$	$(U-L)/(X-L)-1=(U-X)/(X-L)$
$U < X$	0

Chapter 6 Losses and LAE — Loss Trend

[Example 6-8]

- Please determine the trend for each layer if there is a trend of 10% on all claims

		Distribution of Loss Amount by Layer			
			15,000	25,000	50,000
	Original	First	excess of	excess of	excess of
Claim	Loss Amount	10,000	10,000	25,000	50,000
1	5,000	5,000	0	0	0
2	20,000	10,000	10,000	0	0
3	40,000	10,000	15,000	15,000	0
4	70,000	10,000	15,000	25,000	20,000
Total	135,000	35,000	40,000	40,000	20,000

Chapter 6 Losses and LAE — Loss Trend

[Sol]

		Distribution of Loss Amount by Layer			
			15,000	25,000	50,000
	10% of Trend	First	excess of	excess of	excess of
Claim	Loss Amount	10,000	10,000	25,000	50,000
1	5,500	5,500	0	0	0
2	22,000	10,000	12,000	0	0
3	44,000	10,000	15,000	19,000	0
4	77,000	10,000	15,000	25,000	27,000
Total	148,500	35,500	42,000	44,000	27,000
Increase %	10.00%	1.43%	5.00%	10.00%	35.00%

Chapter 6 Losses and LAE — Loss Development vs. Loss Trend

■ Overlap Fallacy

- A double counting of loss development and trend
- Loss development is to reflect the development from occurrence to settlement
- Trend is to reflect the change from the historical occurrence to future occurrence

Chapter 6 Losses and LAE — Loss Development vs. Loss Trend



Chapter 6 Losses and LAE — Loss Adjustment Expense

- The expense occurred by an insurer to close a claim
- Including
 - ALAE (Allocated Loss Adjustment Expense)
 - Claim-related expenses that are directly attributable to a specific claim
 - Example, fees associated with outside legal counsel hired to defend a claim can be directly assigned to a specific claim
 - ULAE (Unallocated Loss Adjustment Expense)
 - Claim-related expenses that cannot be directly assigned to a specific claim
 - Example, salaries of claims department
- Definition has been changed since 1998
 - ALAE → DCC (Defense and Cost Containment)
 - ULAE → A&O (Adjusting and Other)

Chapter 6 Losses and LAE — Loss Adjustment Expense

- Treatment of LAE
 - ALAE
 - In general, ALAE is included in losses, but not always
 - Usually vary by the dollar amount of each claim
 - Incorporated with a % of reported losses
 - ULAE
 - In general, ULAE is usually included in expense
 - Usually vary by the number of claims reported
 - Study how adjusters spend their time

Chapter 6 Losses and LAE — Loss Adjustment Expense

- Simple Approach to incorporate ULAE
 - Assumption
 - ULAE track with loss plus ALAE dollars consistently over time
 - Determine a paid-paid ratio
 - Paid-Paid ratio
= CY paid ULAE / CY paid Losses + ALAE
 - Determine the ULAE
 - Paid-Paid ratio × Trended Ultimate Losses

Chapter 6 Losses and LAE — Loss Adjustment Expense

[Example 6-9]

	(1)	(2)	$\frac{(2)}{(1)}$ (3)
CY	Paid Losses and ALAE	Paid ULAE	ULAE Ratio
2008	913,467	144,026	15.8%
2009	1,068,918	154,170	14.4%
2010	1,234,240	185,968	15.1%
Total	3,216,625	484,164	15.1%

ULAE Factor = 1.151

Chapter 7

Other Expenses and Profit



Chapter 7 Other Expenses and Profit—Underwriting Expenses

- **Commission and Brokerage**
 - Amount paid to insurance agents or brokers as compensation for generating business
 - Typically, paid as a percentage of premium written
- **Other Acquisition**
 - Expenses other than commissions and brokerage expense paid to acquire business
 - Example, advertisements
- **General Expense**
 - The remaining expenses associated with the insurance operations and any other miscellaneous costs
- **Taxes, Licenses, and Fees**
 - All taxes and miscellaneous fees paid, but excluding income taxes

Chapter 7 Other Expenses and Profit—Underwriting Expenses

- Underwriting expenses can be divided into two types
 - Fixed Expense
 - The expenses are the same for each risk or policy, regardless of the size of the premium
 - The expenses is a constant dollar amount for each risk or policy, regardless of the size of the premium
 - Examples
 - Overhead costs
 - Electricity
 - Rental
 - Variable Expense
 - The expenses vary directly with premium
 - The expense is a constant percentage of the premium
 - Examples
 - Premium taxes
 - Commissions

Chapter 7 Other Expenses and Profit—Expense Ratio

- Expense Ratio = Expenses / Premiums
 - The premiums can be either the written premium or the earned premium
 - Written Premium
 - Used when expenses are incurred at the inception of the policy
 - Variable expenses usually use the written premium
 - Earned Premium
 - Used when expenses are incurred throughout the policy
 - It reflects the gradual payment of expenses that can be proportional to the earning of premium over the policy term
 - Fixed expenses usually use the earned premium
 - Impacts if the choice of premiums is not appropriate
 - The impact will be relatively little for a stable company,
 - The expense ratio will be overestimated if the earned premium is used to measure the variable expense for a growing company
 - Earned premium is less than written premium
 - The commissions and brokerage are high

Chapter 7 Other Expenses and Profit—Simple Example

■ Fundament Equation

– Premium = Losses + LAE + UW Expenses + UW Profit

- $P \equiv$ Premium
- $L \equiv$ Losses
- $E_L \equiv$ LAE
- $E_F \equiv$ Fixed Expense
- $V \equiv$ Variable Expense Ratio
- $Q_T \equiv$ UW Profit

– $P = L + E_L + (E_F + V \times P) + Q_T \times P$

– $P = (L + E_L + E_F) / (1 - V - Q_T)$



Chapter 7 Other Expenses and Profit—Simple Example

■ Simple Example

– $L + E_L = 180$

– $E_F = 20$

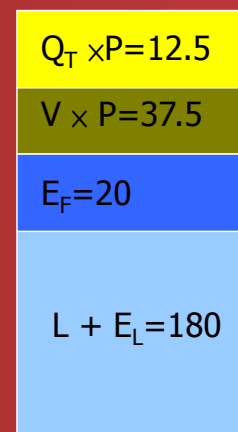
– $V = 15\%$

– $Q_T = 5\%$

➡ $P = (180 + 20) / (1 - 15\% - 5\%) = 250$

– $V = 250 \times 15\% = 37.5$

– $Q_T = 250 \times 5\% = 12.5$



Chapter 7 Other Expenses and Profit—Methods to incorporate expense provisions

- Approaches to derive expense provisions for ratemaking
 - All Variable Expense Method
 - Premium-based Projection Method
 - Exposure/Policy-based Projection Method
- All Variable Expense Method
 - Traditional approach
 - But still widely used
 - Does not differentiate between fixed and variable underwriting expenses
 - Treats all expenses as variable
 - Assumes that expenses ratios during the projected period will be consistent with the historical expenses ratios

Chapter 7 Other Expenses and Profit—Methods to incorporate expense provisions

[Example 7-1]

Items	2013	2014	2015	Total	Selected
(1) Written Premium	1,532,091	1,981,109	2,801,416	6,314,616	
(2) Earned Premium	1,440,166	1,756,600	2,391,263	5,588,028	
(3) Commissions and Brokerage	183,851	198,111	294,149	676,111	
(4) Other Acquisition	72,009	104,707	142,072	318,788	
(5) General Expense	115,213	151,068	215,214	481,494	
(6) Taxes, Licenses, and Fees	30,642	39,622	56,028	126,292	
(7) Commissions and Brokerage Ratio	12.0%	10.0%	10.5%	10.7%	10.5%
(8) Other Acquisition Ratio	4.7%	5.3%	5.1%	5.0%	5.0%
(9) General Expense Ratio	8.0%	8.6%	9.0%	8.6%	8.6%
(10) Taxes, Licenses, and Fees Ratio	2.0%	2.0%	2.0%	2.0%	2.0%
(11) Total Underwriting Expense Ratio	26.7%	25.9%	26.6%	26.4%	26.2%

– Potential distortions

- Underestimate the premium need for a small risk
- Overestimate the premium need for a large risk

[Note] Companies that use this approach may implement a premium discount structure that reduces the expenses loading

Chapter 7 Other Expenses and Profit—Methods to incorporate expense provisions

[Example 7-2]

	(1)	(2)	(3)	$=((1)+(3))/(1-(2))$ (4)	$= (3)/250$ (5)	$= (2)+(5)$ (6)	$= (1)/(1-(6))$ (7)
	Loss Cost	Variable Expense	Fixed Expense	Correct Premium	Fixed Expense Ratio	Total Expense Ratio	All Variable Expense Method
Small Size	135	20.0%	20	194	8.0%	28.0%	188
Medium	180	20.0%	20	250	8.0%	28.0%	250
Large	225	20.0%	20	306	8.0%	28.0%	313

Chapter 7 Other Expenses and Profit—Methods to incorporate expense provisions

■ Premium-based Projection Method

- Schofield, D., “Going From a Pure Premium to a Rate,” CAS Study Note, 1998
 - <http://www.casact.org/library/studynotes/scholfield5.pdf>
- Recognizes the fixed and variable expenses separately
- Like All Variable Expense Method, assumes that expenses ratios during the projected period will be consistent with the historical expenses ratios

Chapter 7 Other Expenses and Profit—Methods to incorporate expense provisions

[Example 7-3]

Items	Total Ratio	Fixed Portion	Expense Ratio	
			Fixed	Variable
(1) Commissions and Brokerage Ratio	10.5%	0.0%	0.0%	10.5%
(2) Other Acquisition Ratio	5.0%	0.0%	0.0%	5.0%
(3) General Expense Ratio	8.6%	75.0%	6.5%	2.2%
(4) Taxes, Licenses, and Fees Ratio	2.0%	0.0%	0.0%	2.0%
(5) Total Underwriting Expense Ratio	26.2%		6.5%	19.7%
(6) Projected Average Premium per Exposure		200		
(7) Projected Fixed Expense per Exposure		13		

– Potential distortions

- Recent rate changes can impact the historical expense ratio
- Significant differences in average premium can distort the fixed expenses

Chapter 7 Other Expenses and Profit—Methods to incorporate expense provisions

■ Exposure/Policy-Based Projection Method

- Diana Childs and Ross Currie, “Expense Allocation in Insurance Ratemaking,” 1980
 - <http://www.casact.org/pubs/dpp/dpp80/80dpp032.pdf>
- Variable expenses are treated the same way as the Premium-Based Projection Method
- Historical fixed expenses are divided by historical exposures or policy count rather than premium

Chapter 7 Other Expenses and Profit—Methods to incorporate expense provisions

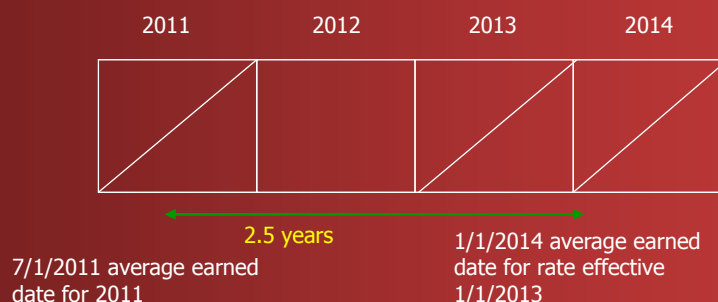
[Example 7-4]

Items	3-Yr Total		3-Yr Expense	
	Expense	Fixed Portion	Fixed	Variable
(1) Commissions and Brokerage Ratio	676,111	0.0%	0	676,111
(2) Other Acquisition Ratio	318,788	0.0%	0	318,788
(3) General Expense Ratio	481,494	75.0%	361,121	120,374
(4) Taxes, Licenses, and Fees Ratio	126,292	0.0%	0	126,292
(5) Total Underwriting Expense Ratio	1,602,685		361,121	1,241,564
(6) 3-Yr Total Earned Exposure		85,000		
(7) Fixed Expense per Exposure		4.25		
(8) 3-Yr Total Written Premium		6,314,616		
(9) Variable Expense Ratio		19.7%		

Chapter 7 Other Expenses and Profit—Trending Expenses

- No need to trend the variable expense
 - Since the variable expenses will automatically change as the premium changes
- Fixed expense should be trended
 - The fixed expense may increase over time due to inflationary pressures
 - Trend from the average date the policies are earned in the historical period to the average earned date in the projection period

[Example 7-5]



Chapter 7 Other Expenses and Profit—Underwriting Profit Provision

- Total Profit = UW Profit + Investment Income
- Underwriting Profit
 - UW Profit = Premium – Losses – LAE – UW Expenses
- Investment Income
 - Including
 - Investment income on capital
 - Investment income on policyholder-supplied funds
 - Investment income on capital
 - Capital funds belong to the owners of the insurance company
 - There is substantial disagreement as to whether this should be returned to insureds
 - Investment income on policyholder-supplied funds
 - Policyholder-supplied funds includes unearned premium reserves and loss reserves
 - Fixed expenses usually use the earned premium

Chapter 7 Other Expenses and Profit—Permissible Loss Ratio

- Variable Permissible Loss Ratio (VPLR)
 - $VPLR = 1 - V - Q_T$
 - The percentage of each premium dollar that is intended to pay for the projected loss and LAE and projected fixed expense
- Permissible Loss Ratio (PLR)
 - $PLR = 1 - F - V - Q_T$
 - The percentage of each premium dollar that is intended to pay for the projected loss and LAE

Chapter 8

Overall Indication



Chapter 8 Overall Indication

- Two basic approaches
 - Pure Premium Method
 - Loss Ratio Method
- Pure Premium Method vs. Loss Ratio Method
 - Mathematically equivalent
 - the results are the same if same data and consistent assumptions are used
 - Differences

Pure Premium Method	Loss Ratio Method
Based on exposure	Based on premium
Existing rate is not required	Existing rate is required
On-Level premium is not required	On-Level premium is required
Produces indicated rates	Produces indicated rate changes

Chapter 8 Overall Indication — Pure Premium Method

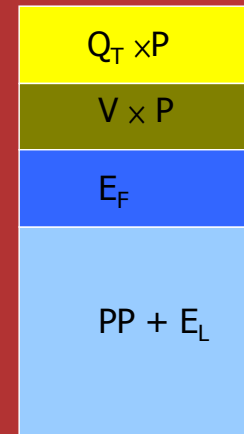
■ Indicated Average Rate Per Exposure

– Rate = Pure Premium + LAE + UW Expenses + Target Profit %

- $R \equiv$ Average Rate per Exposure
- $PP \equiv$ Pure Premium
- $E_L \equiv$ LAE
- $E_F \equiv$ Fixed Expense
- $V \equiv$ Variable Expense Ratio
- $Q_T \equiv$ Target Profit %

– $R = PP + E_L + (E_F + V \times P) + Q_T \times R$

– $R = (PP + E_L + E_F) / (1 - V - Q_T)$



Chapter 8 Overall Indication – Loss Ratio Method

■ Determine the rate change %

– Compare the projected loss ratio (or experience loss ratio, ELR) to the permissible loss ratio (PLR, or target loss Ratio)

- If the projected loss ratio is higher than the permissible loss ratio, the current rate is too low and should be increased
- Indicated Change Factor (A) = ELR / PLR

[Note] The numerator can include ALAE, ULAE, or/and fixed expense, as long as the denominator uses the same definition

■ Determine the rate

– $R = A \times R_0$

Where R : indicated rate

R_0 : current rate

Chapter 8 Overall Indication – Loss Ratio Method

[Example 8-1] Please determine the rate change % using the information below.

- Projected loss and LAE ratio = 65%
- Projected fixed expense ratio = 6.5%
- Variable expense ratio = 25%
- Target profit % = 10%

[Sol 1]

$$\text{PLR} = 1 - 6.5\% - 25\% - 10\% = 58.5\%$$

$$A = \text{ELR} / \text{PLR} = 65\% / 58.5\% = 111.1\%$$

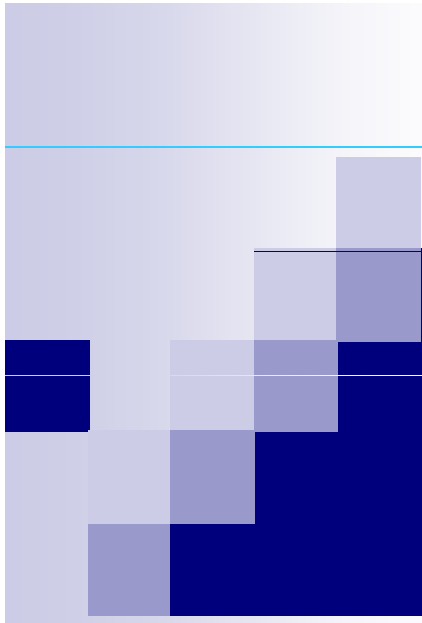
$$\text{Rate change \%} = A - 1 = 11.1\%$$

[Sol 2]

$$\text{VPLR} = 1 - 25\% - 10\% = 65\%$$

$$A = (\text{ELR} + \text{Fixed expense ratio}) / \text{VPLR} \\ = (65\% + 6.5\%) / 65\% = 110\%$$

$$\text{Rate change \%} = 110\% - 1 = 10\%$$



Credibility

鍾孟芳

Kris Chung, FCAS

November 2, 2012

1



Outlines

- Introduction
- Methods for Calculation of Credibility
- Desirable Qualities of a Complement of Credibility
- Methods For Developing Complements of Credibility - *First Dollar Ratemaking*
- Methods For Developing Complements of Credibility - *Excess Ratemaking*

2



Introduction

3



General Concept

- Set rate levels so that rates are “adequate, reasonable, and not unfairly discriminatory”
 - Adequate: Not too low
 - Reasonable: Not too high
 - Not unfairly discriminatory: Allocation of overall rate to individuals is based on cost justification
- At various steps in the ratemaking process, the concept of credibility is introduced
- The credibility of data is commonly denoted by the letter “Z”

4



Definitions of Credibility

- Common vernacular (Webster):
 - “Credibility” = the quality of being credible
 - “Credible” = believable
 - So, credibility is “the quality of being believable”
- In actuarial circles:
 - Credibility is “a measure of the predictive value in a given application that the actuary attaches to a particular body of experience”

-- L.H. Longley-Cook
 - Refers to the degree of believability; a relative concept

5



Why Do We Need Credibility?

- Property / casualty insurance costs (losses) are inherently **stochastic**
 - Losses are fortuitous events
 - Any given insured may or may not have a claim in a given year
 - The size of the claim can vary significantly
 - Data can be viewed as an observation of a result
 - Only one estimate of the “truth” of the probability of having a claim and the distribution of sizes of claims
- So how much can we believe our data?

6



History of Credibility in Ratemaking

- The CAS was founded in 1914, in part to help make rates for a new line of insurance – workers compensation – and credibility was born out the problem of how to blend new experience with initial pricing
- Early pioneers:
 - Mowbray (1914) -- how many trials/results need to be observed before I can believe my data?
 - Albert Whitney (1918) -- focus was on combining existing estimates and new data to derive new estimates:

$$\text{New Rate} = \text{Credibility} * \text{Observed Data} + (1 - \text{Credibility}) * \text{Old Rate}$$
 - Perryman (1932) -- how credible is my data if I have less than required for full credibility?

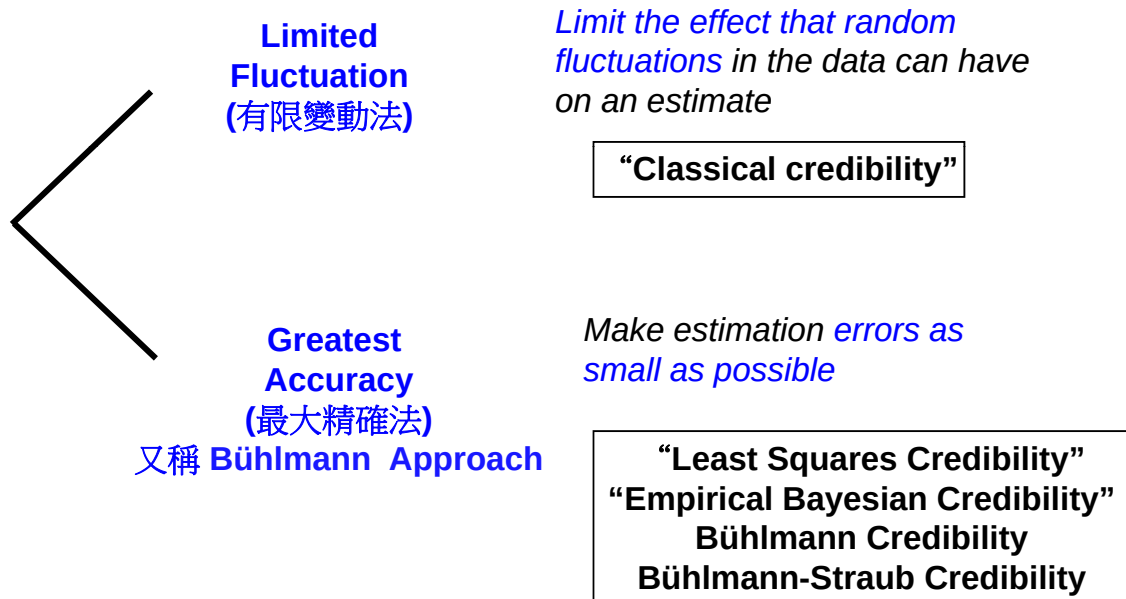
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Methods for Calculation of Credibility

8

Methods of Incorporating Credibility



9

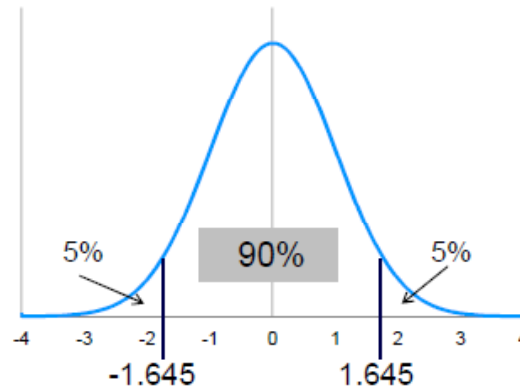
Limited Fluctuation Credibility Description

- “A dependable [estimate] is one for which the probability is high, that it does not differ from the [truth] by more than an arbitrary limit.”
-- Mowbray (1916)
- Alternatively, the credibility Z , of an estimate T , is defined by the probability P , that it within a tolerance $k\%$, of the true value
- Goal: Determine how much data one needs before assigning it with full credibility ($Z = 1$)

10

Limited Fluctuation - Variables

- T: Estimate the data that we want to test for credibility (e.g. loss ratio)
- Z: Credibility, which is between 0 and 1
- k: Tolerance for error (e.g. the observation is within k = 5% of the mean)
- P: Probability that the observation is within k% of the mean
- using the standard Normal distribution (e.g. P = 90% $z_p = 1.645$)



11

Limited Fluctuation Formula - Derivation

- New estimate =
(Credibility)*(Observed Experience) + (1-Credibility)*(Prior Estimate)

$$E2 = Z*T + (1-Z)*E1$$

Add and
subtract
 $Z*E[T]$

$$E2 = Z*T + Z*E[T] - Z*E[T] + (1-Z)*E1$$

regroup

$$E2 = (1-Z)*E1 + Z*E[T] + Z*(T-E[T])$$

Stability

Truth

Random Error

12

Limited Fluctuation Credibility Formula for Z

- The observed experience is considered fully credible when the probability (p) is high that the observed experience will not differ significantly from the expected experience by more than some arbitrary amount (k).

$$\Rightarrow \Pr\{Z(T - E[T]) < kE[T]\} = P$$

$$\text{-or- } \Pr\{T < E[T] + kE[T]/Z\} = P$$

$$\Rightarrow E[T] + kE[T]/Z = E[T] + z_p \text{Var}[T]^{1/2} \\ \text{(assuming } T \sim \text{Normally)}$$

$$\text{-so- } kE[T]/Z = z_p \text{Var}[T]^{1/2}$$

$$\therefore Z = kE[T]/(z_p \text{Var}[T]^{1/2})$$

13

Limited Fluctuation Formula for Z (continued)

- If we assume
 - we are measuring an insurance process that has Poisson frequency, and
 - Severity is constant or severity doesn't matter

- Then $E[T] = \text{number of claims (N)}$, and $E[T] = \text{Var}[T]$, so:

$$Z = kE[T]/z_p \text{Var}[T]^{1/2} \text{ becomes:}$$

$$Z = kE[T]/z_p E[T]^{1/2} = kE[T]^{1/2} / z_p = kN^{1/2} / z_p$$

- Solving for N (# of claims for full credibility, i.e., $Z=1$):

$$N = (z_p/k)^2$$

■ An actuary may regard the loss experience as fully credible if there is a 90% probability that the observed experience is within 5% of its expected value. This is equivalent to saying there is a 95% probability that the observed losses are no more than 5% above the mean. In the Standard Normal table, the 95th percentile is 1.645 standard deviations above the mean; therefore, the expected number of claims needed for full credibility is:

$$E(Y) = \left(\frac{1.645}{0.05} \right)^2 = 1,082.$$

14



Limited Fluctuation– Standards for Full Credibility

- Claim counts required for full credibility based on the previous derivation:

Number of Claims	k			
	2.5%	5.0%	7.5%	10%
P				
90%	4,330	1,082	481	271
95%	6,147	1,537	683	384
99%	10,609	2,652	1,179	663

15



Limited Fluctuation– Standards for Full Credibility

- Exposure counts required for full credibility based on the previous derivation:

P	K	Number of Claims for Full Credibility	Projected Frequency	Number of Exposures for Full Credibility
90%	5%	1,082	10%	10,820
95%	5%	1,537	10%	15,370
99%	5%	2,652	10%	26,520

16

Limited Fluctuation Credibility Formula 2

- Relaxing the assumption that severity doesn't matter,
 - Let "data" = T = aggregate losses = frequency x severity = $N \times S$
 - then $E[T] = E[N]E[S]$
 - and $\text{Var}[T] = E[N]\text{Var}[S] + E[S]^2\text{Var}[N]$
- Plugging these values into the formula
$$Z = kE[T]/z_p\text{Var}[T]^{1/2}$$

and solving for N (@ $Z=1$):

$$N = (z_p/k)^2\{\text{Var}[N]/E[N] + \text{Var}[S]/E[S]^2\}$$

17

Limited Fluctuation Credibility Formula 2

$$N = (z_p/k)^2\{\text{Var}[N]/E[N] + \text{Var}[S]/E[S]^2\}$$

This term is just the full credibility standard derived earlier

Think of this as an adjustment factor to the full credibility standard that accounts for relaxing the assumptions about the data.

The term on the left is derived from the claim frequency distribution and tends to be close to 1 (it is exactly 1 for Poisson).

The term on the right is the square of the c.v. of the severity distribution and can be significant.

18

Limited Fluctuation – Partial Credibility

- Given a full credibility standard based on a number of claims N_{full} , what is the partial credibility of data based on a number of claims N that is less than N_{full} ?
- $Z = (N / N_{full})^{1/2}$
 - Square root rule
 - Designed such that the partial credibility Z will be inversely proportional to the standard deviation of the partially credible data

19

Limited Fluctuation – Increasing Credibility

- Under the square root rule, credibility Z can be increased by
 - Getting more data (increasing N)
 - Accepting a greater margin of error (increasing k)
 - Conceding to smaller P = being less certain (decreasing z_p)

- Based on the formula

$$Z = (N / N_{full})^{1/2}$$

$$Z = [N / (z_p / k)^2]^{1/2}$$

$$Z = k * N^{1/2} / z_p$$

20

Limited Fluctuation – Advantages and Weaknesses

- The strength of limited fluctuation credibility
 - It is the most commonly used and is therefore generally accepted
 - The data required for this approach is readily available
 - The computations are very straightforward
- Weakness of this approach
 - The derivation involves making several simplifying assumptions that may not be true in practice (e.g., no variation in the size of losses)
 - Typical use of the formula based on the Poisson model is inappropriate for most applications
 - Partial credibility formula – the square root rule – only holds for a normal approximation of the underlying distribution of the data but insurance data tends to be skewed

21

Limited Fluctuation – Example 1

- Calculate the loss ratios, given that the expected loss ratio is 75%, and using the square root rule

	Loss Ratio	Claims			
2002	67%	535			
2003	77%	616			
2004	79%	634			
2005	77%	615			
2006	86%	686			
3 year	81%	1,935	Credibility at: 1,082	5,410	Weighted Loss Ratio
5 year	77%	3,086	100%	60%	78.6%
			100%	75%	76.5%
					Indicated Rate Change
					4.8%
					2.0%

Example: $78.6\% = 81\%(0.60) + 75\%(1-0.60)$

Example: $1.020 = 76.5\%/75\% - 1$

22

Limited Fluctuation – Example 2

Given a current territory factor of 1.08, determine the indicated territory factor with 5 years of data. Use the square root rule and the limited fluctuation formula for pure premium. Assume a Poisson frequency distribution and severity coefficient of variation of 1.5.

<u>Year</u>	<u>Territory Exposure</u>	<u>Territory Claim Count</u>	<u>Territory Loss Ratio</u>	<u>Statewide Loss Ratio</u>
2006	3,000	330	125%	78%
2007	3,020	420	153%	83%
2008	3,030	630	269%	85%
2009	3,020	210	122%	79%
2010	3,050	190	108%	72%
'06-'10	15,120	1,780	162%	80%

23

Limited Fluctuation – Example 2

$$N = (z_p/k)^2 \{ \text{Var}[N]/E[N] + \text{Var}[S]/E[S]^2 \}$$

- Remember, with a Poisson distribution, $\text{Var}(N) = E(N)$, so the second term is 1.
- The third term is the square of the coefficient of variation, which is 1.5^2 .
- Now we just need to select the confidence levels.

- $N = (z_p/k)^2 \{ \text{Var}[N]/E[N] + \text{Var}[S]/E[S]^2 \}$

If we want to be within 5% of the true value 90% of the time, the value for $(z/k)^2$ is 1.082 Plugging into the formula:

$$N(\text{claims}) = 1,082 * (1 + 1.5^2) = 3,516.5$$

- Assuming the 5-year statewide frequency is 0.2:

$$N(\text{exposures}) = 3,516.5 / 0.2 = 17,582.5$$

24

Limited Fluctuation – Example 2

- To show the impact of our selection of an exposure standard instead of a claims standard.

<u>Year</u>	<u>Territory Exposure</u>	<u>Territory Claim Count</u>	<u>Exposure Credibility</u>	<u>Claim Credibility</u>
2006	3,000	330	41.3%	30.6%
2007	3,020	420	41.4%	34.6%
2008	3,030	630	41.5%	42.3%
2009	3,020	210	41.4%	24.4%
2010	3,050	190	41.6%	23.2%
'06-'10	15,120	1,780	92.7%	71.1%

Using a claims standard of 3,517 and an exposure standard of 17,583

25

Limited Fluctuation – Example 2

- Determine what the indicated territorial factor, assuming 15% for fixed expenses.

<u>Year</u>	<u>Territory Loss Ratio</u>	<u>Territory Credibility</u>	<u>Statewide Loss Ratio</u>	<u>Cred Wght Loss Ratio</u>
'06-'10	162%	92.7%	80%	156.0%

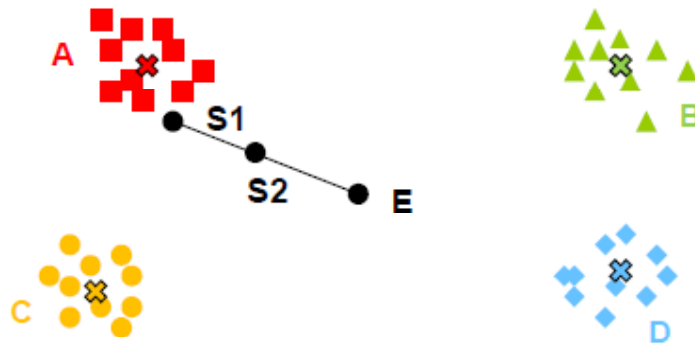
$$156.0\% = 92.7\% \times 162\% + 7.3\% \times 80\%$$

- The final indicated territorial factor is $(156\% / 80\%) \times 0.85 + 0.15 = 1.81$

26

Greatest Accuracy Credibility

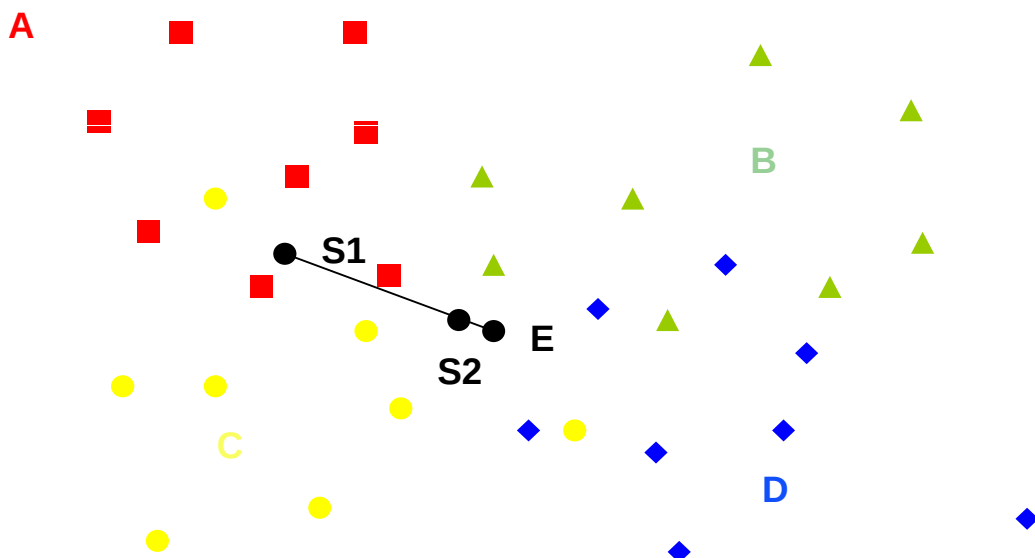
Which data exhibits more credibility?



27

Greatest Accuracy Credibility

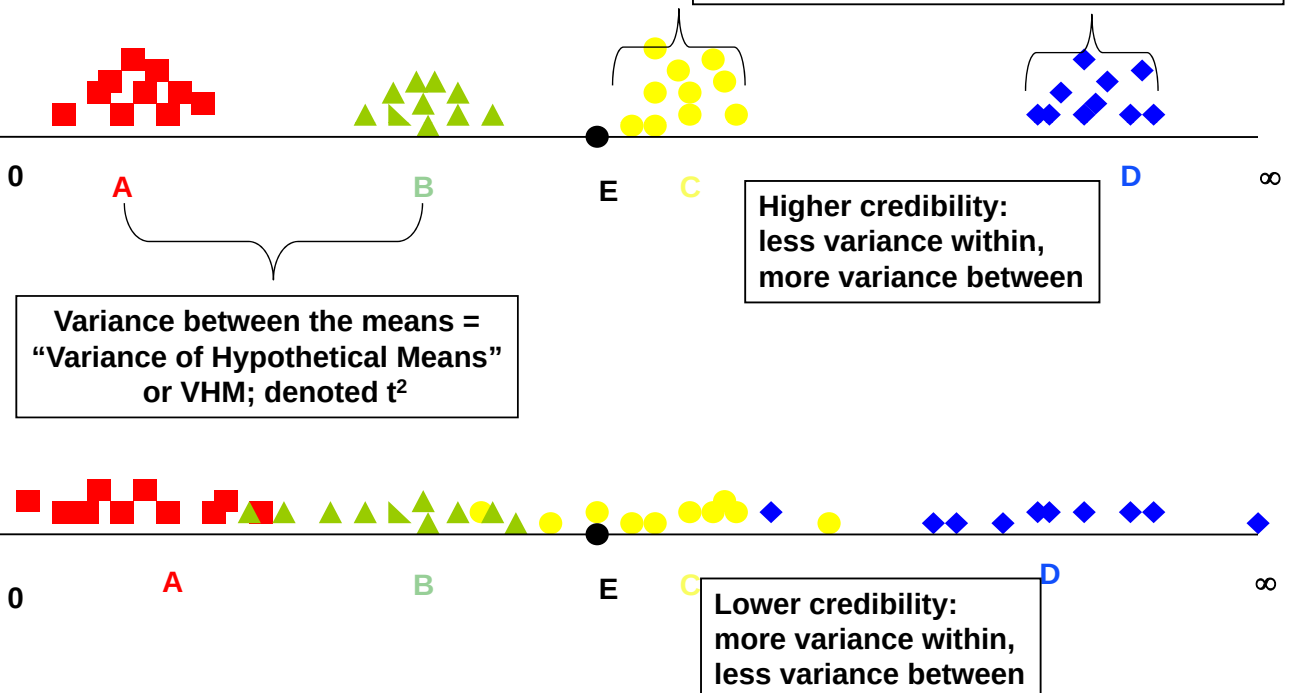
Which data exhibits more credibility?



28

Greatest Accuracy Credibility

Class loss costs per exposure...



29

Bühlmann Credibility

Description

- Bühlmann credibility is commonly referred to as **least squares credibility** since the goal of this approach is to minimize the square of the error between the estimate and the true expected value of the quantity being estimated.

30

Bühlmann Credibility - Derivation

- Suppose you have two independent estimates of a quantity, x and y , with squared errors of u and v respectively
- We wish to weight the two estimates together as our estimator of the quantity:

$$a = zx + (1-z)y$$

- The squared error of a is

$$w = z^2 u + (1-z)^2 v$$

- Find Z that minimizes the squared error of “ a ” – take the derivative of w with respect to z , set it equal to 0, and solve for z :

$$dw/dz = 2zu + 2(z-1)v = 0$$

$$Z = u/(u+v)$$

31

Bühlmann Credibility - Derivation

Using the formula that establishes that the least squares value for Z is proportional to the reciprocal of expected squared errors:

$$Z = (n/s^2)/(n/s^2 + 1/t^2) =$$

$$= n/(n + s^2/t^2)$$

$$= n/(n+k)$$

$$k = s^2/t^2$$

32

Bühlmann Credibility Formula

- New estimate = $(\text{Credibility}) \times (\text{Observed Experience}) + (1 - \text{Credibility}) \times (\text{Prior Mean})$

- $\text{Credibility} = Z = \frac{N}{N + K}$

N: the number of observations

K: expected value of the process variance divided by the variance of the hypothetical means

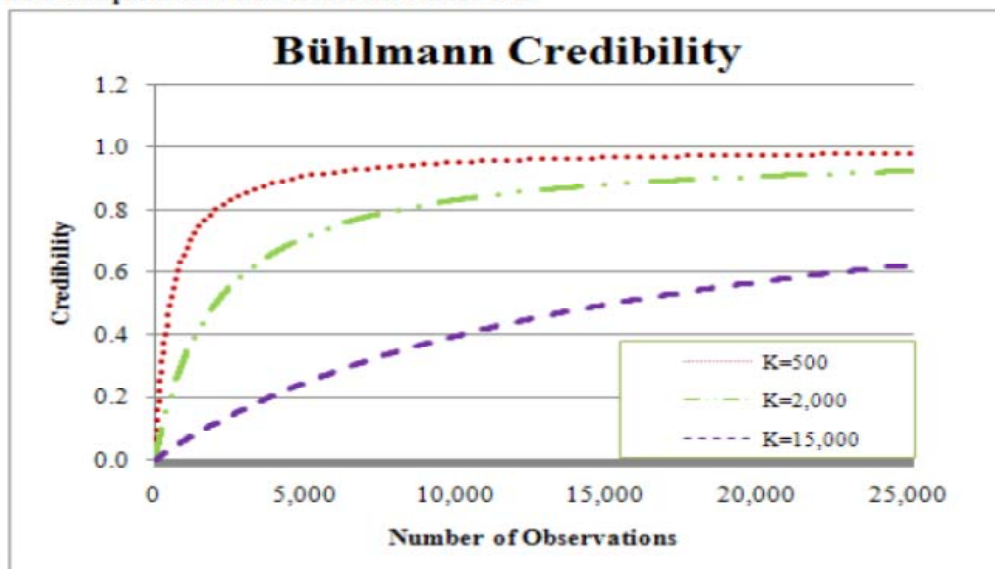
* Ratio K can be described more simply as the ratio of average risk variance and the variance between risks

- Credibility, can be increased by:
 - Increasing n = get more data
 - decreasing s^2 = less variance within classes, e.g., refine data categories
 - increase t^2 = more variance between classes

33

Sensitivity of credibility

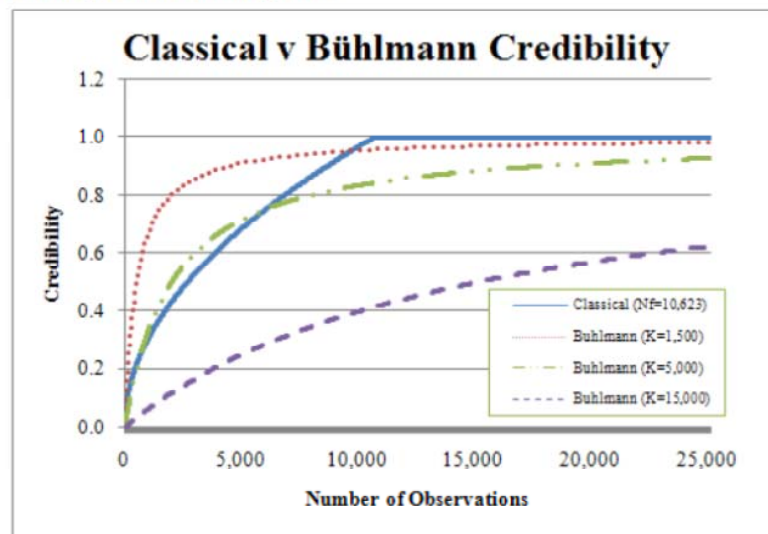
12.2 Comparison of Z for Different Values of K



34

Comparison of Credibility Values

12.3 Comparison of Credibility Values



- For a relatively small number of observations, the Bühlmann credibility estimate is closest to the classical credibility estimate when $K = 5,000$
- As the number of observations gets larger, the Bühlmann credibility estimate is closest to the classical credibility estimate when $K = 1,500$

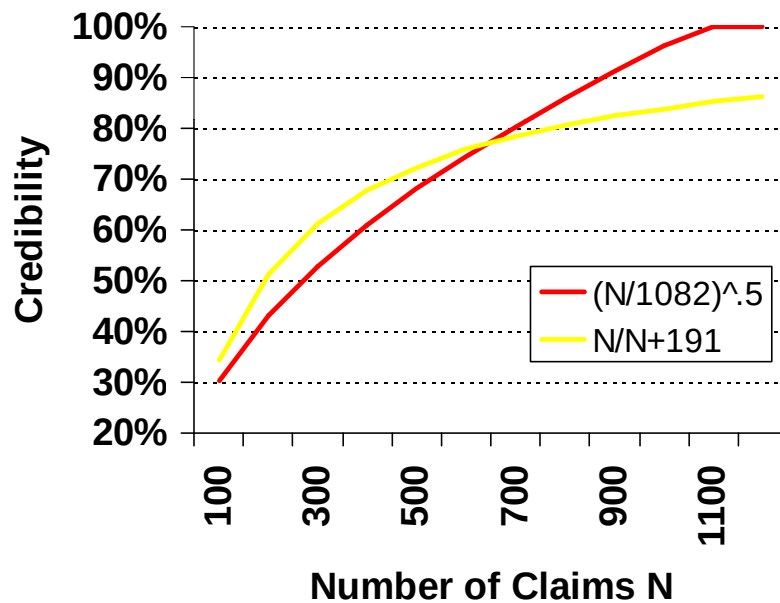
35

Comparison of Credibility Values

- Many practitioners using classical credibility make simplifying assumptions—for example, they ignore the variation in the size of losses and assume that the risks in the subject experience are homogeneous.
- If these same assumptions are made with least squares credibility, then $VHM = 0$ (this is because all of the exposures have exactly the same claim distribution). When $VHM = 0$, then $Z = 0$ and no credibility is assigned to the observed experience.

36

Limited Fluctuation – Partial Credibility



37

Assumptions for Bühlmann Credibility

- $(1-Z)$ is applied to the prior mean
- The risk parameters and risk process do not shift over time
- The expected value of process variance of the sum of N observations increases with N
- The variance of the hypothetical means of the sum of N observations increases with N

38



Bühlmann Credibility - Example

- The observed pure premium is 200 based on 21 observations
- The expected value of the process variance is 2.00
- The variance of hypothetical means is 0.50
- The prior mean is 225

$$K = 2.00 / 0.50 = 4.00$$

$$Z = 21 / (21 + 4.00) = 0.84$$

$$\text{Credibility-weighted Pure Premium} = 0.84 \times 200 + 0.16 \times 225 = 204$$

39



Bühlmann Credibility – Advantages and Weaknesses

- Main advantages
 - Is generally accepted
 - It is based on relative variances or volatility of the data
 - There is no such thing as full credibility
- Main disadvantage of this approach
 - The difficulty of the determination of the expected value of the process variance and the variance of the hypothetical means
 - The Credibility Parameter K, is a property of the *entire* set of data. So, for example, if a data set has a small, volatile class and a large, stable class, the credibility parameter of the two classes would be the same.

40



Desirable Qualities of a Complement of Credibility

41



According to ASOP 25

The actuary should use care in selecting the related experience that is to be blended with the subject experience. Such related experience should have frequency, severity, or other determinable characteristics that may reasonably be expected to be similar to the subject experience. If the proposed related experience does not or cannot be adjusted to meet such criteria, it should not be used. The actuary should apply credibility procedures that appropriately reflect the characteristics of both the subject experience and the related experience

42



Fundamental Principles – What should the actuary consider

- Practical issues
Readily available and easy to calculate
- Competitive market issues
Unbiased and accurate
- Regulatory issues
Logical relationship to base statistic
- Statistical issues
Need to consider all types of error that make up prediction error

43



Desirable qualities for a complement of Credibility

- Accurate
- Unbiased
- Statistically independent from the base statistic
- Available
- Easy to compute
- Logical relationship to base statistic

44



Methods For Developing Complements of Credibility - *First Dollar Ratemaking*

45



Methods for developing complements for first dollar ratemaking

- Loss costs of a larger group that includes the group being rated
- Loss costs of a larger related group
- Rate change from the larger group applied to present rates
- Harwayne's method
- Trended present rates
- Competitor's rates

46

Loss costs of a larger group that includes the group being rated

- Complements of credibility for Group A, Class 1

12.4 Data from a Larger Group

Rate Group	Class	Latest Year		Latest 3 Years	
		Exposures	Pure Premium	Exposures	Pure Premium
A	1	100	\$ 50	250	\$ 64
	2	300	\$ 67	850	\$ 65
	3	400	\$ 48	1,100	\$ 50
	Subtotal	800	\$ 55	2,200	\$ 57
B	Subtotal	600	\$ 48	1,700	\$ 32
C	Subtotal	1,000	\$ 72	2,800	\$ 86
D	Subtotal	1,600	\$ 94	5,600	\$ 87
Total	Total	4,000	\$ 74	12,300	\$ 74

47

Loss costs of a larger group that includes the group being rated

Evaluation

- The complement is based on a greater volume of data so it's likely to have a lower process variance than the subject experience
- If the complement excludes the subject experience, then it's likely to be independent
- If the subject experience is included, it should not dominate the group
- It's available and easy to calculate

48



Loss costs of a larger related group

Method Description

- Use the loss costs of a separate but similar, large group
- For example, a homeowners insurer may use the contents loss experience from the owners forms to supplement the contents experience for the condos form

Evaluation

- It is generally biased though the magnitude and direction of bias are unknown
- If the actuary can adjust the related experience to match the exposure to loss in the subject experience, the bias can be reduced
- In the example mentioned above, the actuary needs to consider how the exposure to loss for condos differs from owned homes and adjust the experience accordingly
- Since the complement does not contain the subject experience, this lack of dependence may make it a better choice than the first method described

49



Rate Change from the Larger Group Applied to Present Rates

Method Description

- Using the rate change indicated for a larger group applied to the current loss cost of the subject experience, rather than using the larger group's loss costs directly to reduce the bias

$$C = \text{Current Loss Cost of Subject Experience} \times \left(\frac{\text{Larger Group Indicated Loss Cost}}{\text{Larger Group Current Average Loss Cost}} \right)$$

Evaluation

- Assuming the rate changes are relatively small, this complement is likely to be accurate over the long term
- The data for this method is most likely readily available, and the calculations are very straightforward

50

Harwayne's Method

Method Description

- Harwayne's Method is used when the subject experience and related experience have significantly different distributions, and the related experience requires adjustment before it can be blended with the subject experience.
- In the application of Harwayne's method, the complement of credibility is determined using countrywide data (excluding the base state being reviewed), but the countrywide data is adjusted to remove overall differences between states.

51

Harwayne's Method

12.5 Harwayne Method

State	Class	Exposure	Losses	Pure Premium
A	1	100	\$ 250	2.50
	2	125	\$ 500	4.00
	Subtotal	225	\$ 750	3.33
B	1	190	\$ 600	3.16
	2	325	\$ 1,500	4.62
	Subtotal	515	\$ 2,100	4.08
C	1	180	\$ 500	2.78
	2	450	\$ 1,800	4.00
	Subtotal	630	\$ 2,300	3.65
All	1	470	\$ 1,350	2.87
	2	900	\$ 3,800	4.22
	Total	1,370	\$ 5,150	3.76

- The following example illustrates the steps necessary to calculate the complement for class 1 of state A.

- First Step, calculate average pure premium for state A, B, C

$$\bar{L}_A = \frac{100 \times 2.50 + 125 \times 4.00}{100 + 125} = 3.33.$$

$$\hat{L}_B = \frac{100 \times 3.16 + 125 \times 4.62}{100 + 125} = 3.97,$$

$$\hat{L}_C = \frac{100 \times 2.78 + 125 \times 4.00}{100 + 125} = 3.46.$$

- Adjustment factors are calculated by dividing the average pure premium for state A by the reweighted average pure premium for B and C:

$$F_B = \frac{\bar{L}_A}{\hat{L}_B} = \frac{3.33}{3.97} = 0.84, \quad F_C = \frac{\bar{L}_A}{\hat{L}_C} = \frac{3.33}{3.46} = 0.96.$$

- The adjusted loss costs for class 1 in states B and C,

$$\hat{L}_{1,B} = \bar{L}_{1,B} \times F_B = 3.16 \times 0.84 = 2.65,$$

$$\hat{L}_{1,C} = \bar{L}_{1,C} \times F_C = 2.78 \times 0.96 = 2.67.$$

Complement of credibility for class 1 of state A

$$C = \frac{\hat{L}_{1,B} \times X_{1,B} + \hat{L}_{1,C} \times X_{1,C}}{X_{1,B} + X_{1,C}} = \frac{2.65 \times 190 + 2.67 \times 180}{190 + 180} = 2.66.$$

52



Harwayne's Method

Evaluation

- The complement derived from this method is unbiased as it adjusts for the distributional differences.
- The use of multi-state data generally implies the complement is reasonably accurate as long as there is sufficient countrywide data to minimize the process variance.
- Since the subject experience and related experience consider data from different states, the complement is considered mostly independent.
- The data for the complement is usually available but the computations can be time-consuming and complicated.

53



Trended Present Rates

Method Description

- Used when there is no larger group to use for the complement.
- Actuaries may rely on the current rates as the best available proxy for the indicated rate but two adjustments are made before using the current rates.
- First, insurers do not always implement the rate that is indicated. Thus, the current rates should be adjusted to what was previously indicated rather than what was implemented.
- Second, changes in loss cost level may have occurred between the time the current rates were implemented and the time of the review. Sources of such changes may be monetary inflation, distributional shifts, etc.

The actuary should select an appropriate annual loss trend and apply it from the original target effective date of the current rates to the target effective date of the new rates.

$$C = \text{Present Rate} \times \text{Loss Trend Factor} \times \frac{\text{Prior Indicated Loss Cost}}{\text{Loss Cost Implemented with Last Review}}$$

54



Trended Present Rates

■ Example

- Present average rate is \$200
- The selected annual loss trend is 5%
- The rate change indicated in the last review was 10%, and the target effective date was January 1, 2011
- The rate change implemented with the last review was 6%, and the actual effective date was February 1, 2011
- The proposed effective date of the next rate change is January 1, 2013

■ First, the loss trend length must be measured

The length from the target effective date of the last rate review (January 1, 2011) to the target effective date of the next rate change (January 1, 2013), or two years.

■ The complement of Credibility

$$C = \$200 \times (1.05)^2 \times \frac{1.10}{1.06} = \$229.$$

55



Trended Present Rates

Evaluation

- The accuracy of this complement depends largely on the process variance of the historical loss costs. That is why it is used primarily for indications with voluminous data.
- This complement is unbiased since pure trended loss costs are unbiased (with no updating for more current loss costs).
- This complement may or may not be independent depending on the historical experience used to determine the subject experience and complement.
For example, if the complement comes from a review that used data from years 2007 through 2010, and the subject experience is based on data from 2008 through 2011, then the two are not independent.
- The data required is readily available, the calculations are very straightforward, and the approach is easily explainable.

56



Competitors' Rates

Method Description

- New or small companies with small volumes of data often find their own data too unreliable for ratemaking.
- The actuary may use the competitors' rates as a complement.
- The rationalization is that if the competitors have a much larger number of exposures, the competitors' statistics have less process error.

Evaluation

- An actuary must consider that competitors' manual rates are not only based on the competitors' loss costs but also reflect marketing considerations, judgment, and the effects of the regulatory.
- Competitors may also have different underwriting and claim practices than the subject company, which creates bias that may be difficult to quantify.
- The competitors' rates will be independent of the company data.
- While the calculations may be straightforward, the data needed for this complement may be difficult or time-consuming to obtain.
- Even with the potential differences between competitors, the rates of a similar competitor bear a logical relationship and are generally accepted as a complement by regulators.

57



Methods For Developing Complements of Credibility - *Excess Ratemaking*

58



Methods for developing complements for excess ratemaking

- Increased limits analysis
- Lower limits analysis
- Limit analysis

59



Increased limits analysis

Method Description

- Actuaries use this method when data is available for ground-up losses through the attachment point (i.e., losses have not been truncated at any point below the bottom of the excess layer being priced).
- Increased limits factors are used to adjust losses capped at the attachment point to produce an estimate of losses in the specific excess layer.

■

$$C = L_A \times \left(\frac{\text{ILF}_{A+L} - \text{ILF}_A}{\text{ILF}_A} \right) = L_A \times \left(\frac{\text{ILF}_{A+L}}{\text{ILF}_A} - 1.0 \right).$$

- L_A are the losses capped at the attachment point A ;
- ILF_A is the increased limits factor for the attachment point A ;
- ILF_{A+L} is the increased limits factor for the sum of the attachment point A and the excess insurer's limit of liability L .

60

Increased limits analysis

Calculate the complement of credibility for the excess layer between \$500,000 and \$750,000 (i.e., \$250,000 of coverage in excess of \$500,000).

- Assume the losses capped at \$500,000 are \$2,000,000
- The following increased limits factors apply:

12.6 ILFs

Limit of Liability	Increased Limits Factor
\$ 100,000	1.00
\$ 250,000	1.75
\$ 500,000	2.50
\$ 750,000	3.00
\$ 1,000,000	3.40

$$C = \$2,000,000 \times \left(\frac{3.00}{2.50} - 1.0 \right) = 400,000.$$

61

Increased limits analysis

Evaluation

- If the subject experience has a different size of loss distribution than that used to develop the increased limits factors, then the results of this procedure will be biased. This is particularly relevant as the increased limits factors may be based on industry data rather than the insurer's own data.
- The error with this approach is not the process error but rather the parameter error associated with the selection of the increased limits factors. Thus, the error associated with this estimate tends to be independent of the error associated with the base statistic.
- This procedure requires increased limits factors—preferably industry factors—and ground-up losses that have not been truncated below the attachment point. To the extent that information is available, the procedure is practical. In terms of acceptability, however, this estimate is more logically related to the data below the attachment point (which is used for the projection) than to the data in the layer, and this may be controversial.

62

Lower limits analysis

Method Description

- If those losses are too sparse to be reliable, the actuary may prefer to use losses capped at a limit lower than the attachment point. This lower limit can often be the basic limit.

- $$C = L_d \times \left(\frac{\text{ILF}_{A+L} - \text{ILF}_A}{\text{ILF}_d} \right),$$

- L_d are the losses capped at the lower limit, d ;
- ILF_A is the increased limits factor for the attachment point A ;
- ILF_d is the increased limits factor for the lower limit, d .
- ILF_{A+L} is the increased limits factor for the sum of the attachment point A and the excess insurer's limit of liability L (i.e., this sum represents the top of the excess layer being priced).

63

Lower limits analysis

Calculate the complement of credibility for the excess layer between \$500,000 and \$750,000 (i.e., \$250,000 of coverage in excess of \$500,000).

- Assume the losses capped at \$250,000 are \$1,500,000
- The following increased limits factors apply:

12.6 ILFs

Limit of Liability	Increased Limits Factor
\$ 100,000	1.00
\$ 250,000	1.75
\$ 500,000	2.50
\$ 750,000	3.00
\$ 1,000,000	3.40

$$C = \$1,500,000 \times \left(\frac{3.00 - 2.50}{1.75} \right) = \$428,571.$$

64

Lower limits analysis

Evaluation

- It is difficult to determine whether this is more or less accurate than the previously discussed complement.
- Intuitively, this complement will be more biased as the differences in size of loss distributions will be exacerbated when using losses truncated at lower levels.
- On the other hand, using losses capped at lower limits may increase the stability of the estimate.
- Like the previous complement, the error associated with this complement is generally independent of the error of the base statistic.
- Insurers generally code losses capped at basic limits for statistical reporting purposes. If some other lower limit is chosen, the data may not be as available. The calculations are no more difficult than the previously discussed complement, but the criticism of being logically related.

65

Limits analysis

Method Description

- Primary insurers generally sell policies with a wide variety of policy limits. Some of the individual policy limits fall below the attachment point, and some extend beyond the top of the excess layer.
- For this approach the actuary analyzes the policies at each limit of coverage separately. The actuary calculates the estimated losses in a given layer using the premium volume and expected loss ratio in that layer. Then the actuary performs an increased limits factor analysis on each first dollar limit's loss costs separately.

$$C = LR \times \sum_{d \geq A} P_d \times \frac{(\text{ILF}_{\min(d, A+L)} - \text{ILF}_A)}{\text{ILF}_d},$$

- LR = Total limits loss ratio,
- P_d = Total premium for policies with limit d
- The ILFs have the same meaning as previously discussed.

66

Limits analysis

12.7 Example Calculation for Limits Approach

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Limit of Liability (<i>d</i>)	Premium	Expected Loss Ratio	Expected Total Losses	ILF @ <i>d</i>	ILF @ <i>A</i>	ILF @ <i>A+L</i>	% Loss In Layer	Expected Loss in Layer
\$ 100,000	\$ 1,000,000	60.0%	\$ 600,000	1.00	2.50	3.00	0.0%	\$ -
\$ 250,000	\$ 500,000	60.0%	\$ 300,000	1.75	2.50	3.00	0.0%	\$ -
\$ 500,000	\$ 200,000	60.0%	\$ 120,000	2.50	2.50	3.00	0.0%	\$ -
\$ 750,000	\$ 200,000	60.0%	\$ 120,000	3.00	2.50	3.00	16.7%	\$ 20,040
\$ 1,000,000	\$ 75,000	60.0%	\$ 45,000	3.40	2.50	3.00	14.7%	\$ 6,615
Total	\$ 1,975,000		\$ 1,185,000					\$ 26,655

$$\begin{aligned}
 (4) &= (2) \times (3) \\
 (8) &= \begin{cases} \text{if } d \leq A \text{ then } 0.0\% \\ \text{if } A < d \leq A+L \text{ then } [(5) - (6)] / (5) \\ \text{if } d > A+L \text{ then } [(7) - (6)] / (5) \end{cases} \\
 (9) &= (4) \times (8)
 \end{aligned}$$

67

Limits analysis

Evaluation

- This complement is biased and inaccurate to the same extent as the prior two complements.
- It involves relying on the additional assumption that the expected loss ratio does not vary by limit.
- Because this type of excess loss analysis is typically undertaken by reinsurers that do not have access to the full loss distribution, this may be the only method available.
- It is more time-consuming to compute, but the calculations are straightforward.
- It generates the same criticism as the other methods because it is not based on actual data from the layer being priced.

68

Fitted Curves

Method Description

- Actual loss distributions can be very volatile, especially in the tail of the distribution (i.e., the higher losses).
- Actuaries may fit curves to the actual data to smooth out the volatility and to extrapolate the distribution to higher limits.
- Once the curve is fitted, the techniques described in Chapter 11 can be used to determine the expected losses in the layer being priced.
- The following formula is used to determine the percentage of the curve's total losses that are expected in the excess layer:

$$\% \text{ Losses in Layer } (A, A+L) = \frac{\int_A^{A+L} (x-A)f(x)dx + \int_{A+L}^{\infty} Lf(x)dx}{\int_{-\infty}^{\infty} xf(x)dx}.$$

69

Fitted Curves

Evaluation

- This complement tends to be less biased and more stable than the other excess methods, assuming that the fitted curve replicates the general shape of the actual data.
- This approach tends to be significantly more accurate than the others when there are relatively few claims in the higher layers.
- Because the curve-fitting process involves the underlying data, it can be heavily dependent on the existence or non-existence of larger claims. Thus, the error associated with a complement developed using this approach will tend to be less independent than complements determined from the other approaches.
- This approach tends to be the most computationally complex and requires data that may not be readily available.
- The complement developed using this approach tends to be the most logically related to the losses in the layer than the others as the data is more fully used.
- However, the computational complexity may make it difficult to communicate.

70



Werner, G, and Modlin, C., Basic Ratemaking

Chapter 16: Claims-Made Ratemaking

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2011.11.06



CONTENT

- Introduction
- Aggregation of losses by report year and report year lag
- Coverage triggers for claims-made coverage
- The five principles of claims-made policies
- Issues related to coordinating coverage between claims-made and occurrence policies



INTRODUCTION

- Loss trends for liability lines during the 1960s and 1970s
 - increased dramatically in severity due to high **economic and social inflation**
 - as well as increases in claim frequency
 - especially the case for professional liability insurance including medical malpractice
 - It took several years before insurers realized that their products were significantly underpriced because of the long-tailed nature of professional liability
 - This delay in recognizing price inadequacy highlights the significant pricing risk that exists for long-tailed insurance



INTRODUCTION

- The long period between the occurrence of a claim and the settlement of a claim can be driven by
 - **Reporting Lag** :
the time between the occurrence date and report date
 - relates to **pure IBNR** – claims that are incurred but not reported
 - Ex.: it may take several years for the physician's error to cause identifiable symptoms
 - **Settlement Lag** :
the time between the report date and settlement date
 - relates to **IBNER** – claims that are incurred but not enough reported
 - Ex.: it may take many years for the claim to be ultimately settled due to factors such as the need for ongoing treatment and lengthy court proceedings



COVERAGE TRIGGERS

- The major difference between claims-made and occurrence coverage is that the coverage trigger
 - For **claims-made policies**
 - The coverage trigger is the date the claim is reported
 - Only needs to project claims reported during next year's policy period
 - For **occurrence policies**
 - The coverage trigger is the date the event occurs
 - Must consider claims that will be reported many years into the future



REPORT YEAR AGGREGATION

- Categorizes claims by the year reported and the report lag:

Report Year	Report Lag				
	0	1	2	3	4
2010	L(2010,0)	L(2010,1)	L(2010,2)	L(2010,3)	L(2010,4)
2011	L(2011,0)	L(2011,1)	L(2011,2)	L(2011,3)	L(2011,4)
2012	L(2012,0)	L(2012,1)	L(2012,2)	L(2012,3)	L(2012,4)
2013	L(2013,0)	L(2013,1)	L(2013,2)	L(2013,3)	L(2013,4)
2014	L(2014,0)	L(2014,1)	L(2014,2)	L(2014,3)	L(2014,4)
2015	L(2015,0)	L(2015,1)	L(2015,2)	L(2015,3)	L(2015,4)

- L(2010,0) : a claim that occurs in 2010 and is reported in year 2010
- L(2012,2) : a claim that is reported in 2012 after a report lag of two year



REPORT YEAR AGGREGATION

- **Row**
 - claims reported in a given year (i.e., the report year)
- **Column**
 - claims that share the same reporting lag
- **Diagonal** (top left to bottom right)
 - claims that occurred in the same year (i.e., the same accident year)
- **Occurrence policies** $(Y) = \sum_0^N L(Y+i, i)$
 - Occurrence policies (2010) = $L(2010,0) + L(2011,1) + \dots + L(2014,4)$
- **Claims-made policies** $(Y) = \sum_0^N L(Y, i)$
 - Occurrence policies (2010) = $L(2010,0) + L(2010,1) + \dots + L(2010,4)$



REPORT YEAR AGGREGATION

- 2010 Claims-Made Policies:

Report Year	Report Lag				
	0	1	2	3	4
2010	L(2010,0)	L(2010,1)	L(2010,2)	L(2010,3)	L(2010,4)
2011	L(2011,0)	L(2011,1)	L(2011,2)	L(2011,3)	L(2011,4)
2012	L(2012,0)	L(2012,1)	L(2012,2)	L(2012,3)	L(2012,4)
2013	L(2013,0)	L(2013,1)	L(2013,2)	L(2013,3)	L(2013,4)
2014	L(2014,0)	L(2014,1)	L(2014,2)	L(2014,3)	L(2014,4)
2015	L(2015,0)	L(2015,1)	L(2015,2)	L(2015,3)	L(2015,4)



REPORT YEAR AGGREGATION

- 2010 Occurrence Policies:

Report Year	Report Lag				
	0	1	2	3	4
2010	L(2010,0)	L(2010,1)	L(2010,2)	L(2010,3)	L(2010,4)
2011	L(2011,0)	L(2011,1)	L(2011,2)	L(2011,3)	L(2011,4)
2012	L(2012,0)	L(2012,1)	L(2012,2)	L(2012,3)	L(2012,4)
2013	L(2013,0)	L(2013,1)	L(2013,2)	L(2013,3)	L(2013,4)
2014	L(2014,0)	L(2014,1)	L(2014,2)	L(2014,3)	L(2014,4)
2015	L(2015,0)	L(2015,1)	L(2015,2)	L(2015,3)	L(2015,4)



PRINCIPLES OF CLAIMS-MADE POLICIES

- A claims-made policy should always cost less than an occurrence policy as long as claim costs are increasing
- If there is a sudden, unpredictable change in the underlying trends, the claims-made policy priced based on the prior trend will be closer to the correct price than an occurrence policy based on the prior trend
- If there is a sudden, unexpected shift in the reporting pattern, the cost of a mature claims-made policy (i.e., a policy that covers claims reported during the policy period regardless of accident date) will be affected relatively little, if at all, relative to the occurrence policy
- Claims-made policies incur no liability for IBNR, so the risk of reserve inadequacy is greatly reduced
- The investment income earned from claims-made policies is substantially less than under occurrence policies



PRINCIPLES OF CLAIMS-MADE POLICIES

- Example
 - Assumptions
 - Exposure levels are constant
 - The average loss cost for Report Year 2010 is \$1,000
 - Loss costs increase by 5% each report year
 - An equal number of incurred claims are reported each year and all claims are reported within five years of occurrence (i.e., 20% reported each year)
 - Loss costs do not vary by report year lag. Also, any trends affecting settlement lag have been ignored

11



PRINCIPLES OF CLAIMS-MADE POLICIES

- Example
 - Assumptions

Report Year	Report Lag				
	0	1	2	3	4
2010	200.00	200.00	200.00	200.00	200.00
2011	210.00	210.00	210.00	210.00	210.00
2012	220.50	220.50	220.50	220.50	220.50
2013	231.53	231.53	231.53	231.53	231.53
2014	243.10	243.10	243.10	243.10	243.10
2015	255.26	255.26	255.26	255.26	255.26
2016	268.02	268.02	268.02	268.02	268.02
2017	281.42	281.42	281.42	281.42	281.42
2018	295.49	295.49	295.49	295.49	295.49

+5%

12



PRINCIPLES OF CLAIMS-MADE POLICIES

- Example
 - Claims-Made Loss Costs

Report Year	Report Lag					Claims-made Loss Costs
	0	1	2	3	4	
2010	200.00	200.00	200.00	200.00	200.00	\$1,000.00
2011	210.00	210.00	210.00	210.00	210.00	\$1,050.00
2012	220.50	220.50	220.50	220.50	220.50	\$1,102.50
2013	231.53	231.53	231.53	231.53	231.53	\$1,157.65
2014	243.10	243.10	243.10	243.10	243.10	\$1,215.50
2015	255.26	255.26	255.26	255.26	255.26	\$1,276.30
2016	268.02	268.02	268.02	268.02	268.02	\$1,340.10
2017	281.42	281.42	281.42	281.42	281.42	\$1,407.10
2018	295.49	295.49	295.49	295.49	295.49	\$1,477.45

13



PRINCIPLES OF CLAIMS-MADE POLICIES

- Example
 - Occurrence Loss Costs

Report Year	Report Lag					Occurrence Loss Costs
	0	1	2	3	4	
2010	200.00	200.00	200.00	200.00	200.00	
2011	210.00	210.00	210.00	210.00	210.00	
2012	220.50	220.50	220.50	220.50	220.50	
2013	231.53	231.53	231.53	231.53	231.53	
2014	243.10	243.10	243.10	243.10	243.10	\$1,105.13
2015	255.26	255.26	255.26	255.26	255.26	\$1,160.39
2016	268.02	268.02	268.02	268.02	268.02	\$1,218.41
2017	281.42	281.42	281.42	281.42	281.42	\$1,279.33
2018	295.49	295.49	295.49	295.49	295.49	\$1,343.29

14



PRINCIPLES OF CLAIMS-MADE POLICIES

- **Principle 1 :**

A claims-made policy should always cost less than an occurrence policy as long as claim costs are increasing

15



PRINCIPLES OF CLAIMS-MADE POLICIES

- **Principle 1 : (conti.)**

- Occurrence policies require the actuary to make projections about the settlement of claims that occur further out into the future
 - For a 2011 occurrence policy, it is need to project the ultimate value of claims that occur in 2011 and may not even be reported until 2015
 - For a 2011 claims-made policy, it only needs to project the ultimate cost of claims that will be reported in that year
 - For claims-made policies, there is a shorter period of time between coverage trigger and settlement date

16



PRINCIPLES OF CLAIMS-MADE POLICIES

- **Principle 2 :**

If there is a sudden, unpredictable change in the underlying trends, the claims-made policy priced based on the prior trend will be closer to the correct price than an occurrence policy based on the prior trend

17



PRINCIPLES OF CLAIMS-MADE POLICIES

- **Principle 2 : (conti.)**

- **Example:**

Restates the above example assuming the actual loss cost trend by report year is 7% instead of 5%

Report Year	Report Lag				
	0	1	2	3	4
2010	200.00	200.00	200.00	200.00	200.00
2011	214.00	214.00	214.00	214.00	214.00
2012	228.98	228.98	228.98	228.98	228.98
2015	245.01	245.01	245.01	245.01	245.01
⋮	⋮	⋮	⋮	⋮	⋮

+7%

18



PRINCIPLES OF CLAIMS-MADE POLICIES

■ Principle 2 : (conti.)

□ Claims-Made Loss Costs

Report Year	Report Lag					Claims-made Loss Costs
	0	1	2	3	4	
2010	200.00	200.00	200.00	200.00	200.00	\$1,000.00
2011	214.00	214.00	214.00	214.00	214.00	\$1,070.00
2012	228.98	228.98	228.98	228.98	228.98	\$1,144.90
2013	245.01	245.01	245.01	245.01	245.01	\$1,225.05
2014	262.16	262.16	262.16	262.16	262.16	\$1,310.80
2015	280.51	280.51	280.51	280.51	280.51	\$1,402.55
2016	300.15	300.15	300.15	300.15	300.15	\$1,500.75
2017	321.16	321.16	321.16	321.16	321.16	\$1,605.78
2018	343.64	343.64	343.64	343.64	343.64	\$1,718.20

19



PRINCIPLES OF CLAIMS-MADE POLICIES

■ Principle 2 : (conti.)

□ Occurrence Loss Costs

Report Year	Report Lag					Occurrence Loss Costs
	0	1	2	3	4	
2010	200.00	200.00	200.00	200.00	200.00	
2011	214.00	214.00	214.00	214.00	214.00	
2012	228.98	228.98	228.98	228.98	228.98	
2013	245.01	245.01	245.01	245.01	245.01	
2014	262.16	262.16	262.16	262.16	262.16	\$1,150.15
2015	280.51	280.51	280.51	280.51	280.51	\$1,230.66
2016	300.15	300.15	300.15	300.15	300.15	\$1,316.81
2017	321.16	321.16	321.16	321.16	321.16	\$1,408.99
2018	343.64	343.64	343.64	343.64	343.64	\$1,507.62

20



PRINCIPLES OF CLAIMS-MADE POLICIES

■ Principle 2 : (conti.)

- The unexpected increase in trend resulted in the Report Year 2011 loss cost for the claims-made policy is 1.9%
(= $\$1,070.00 / \$1,050.00 - 1.0$)
- The unexpected increase in trend resulted in the Accident Year 2011 loss cost for the occurrence policy is 6.1%
(= $\$1,230.66 / \$1,160.39 - 1.0$)
- An error made in the trend selection for occurrence policies has more of an impact than for claims-made policies



PRINCIPLES OF CLAIMS-MADE POLICIES

■ Principle 3 :

If there is a sudden, unexpected shift in the reporting pattern, the cost of a mature claims-made policy (i.e., a policy that covers claims reported during the policy period regardless of accident date) will be affected relatively little, if at all, relative to the occurrence policy



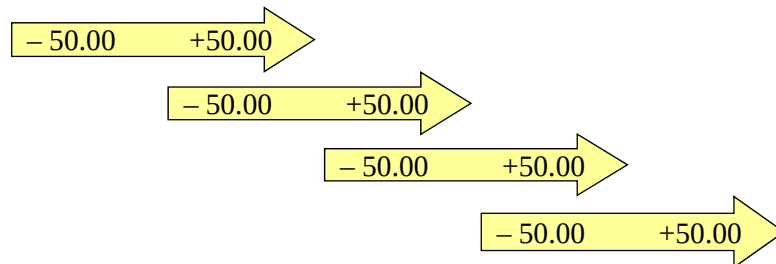
PRINCIPLES OF CLAIMS-MADE POLICIES

■ Principle 3 : (conti.)

□ Example:

Instead of 20% of the claims being reported each year, assume that 5% of the claims are reported one year later than expected, but all claims are reported within five years (\$50 = \$1,000 × 5%)

Report Year	Report Lag				
	0	1	2	3	4
2010	150.00	200.00	200.00	200.00	250.00



PRINCIPLES OF CLAIMS-MADE POLICIES

■ Principle 3 : (conti.)

□ Claims-Made Loss Costs

Report Year	Report Lag					Claims-made Loss Costs
	0	1	2	3	4	
2010	150.00	200.00	200.00	200.00	250.00	\$1,000.00
2011	157.50	210.00	210.00	210.00	262.50	\$1,050.00
2012	165.38	220.50	220.50	220.50	275.63	\$1,102.51
2013	173.64	231.53	231.53	231.53	289.41	\$1,157.64
2014	182.33	243.10	243.10	243.10	303.88	\$1,215.51
2015	191.44	255.26	255.26	255.26	319.07	\$1,276.29
2016	201.02	268.02	268.02	268.02	335.03	\$1,340.11
2017	211.07	281.42	281.42	281.42	351.78	\$1,407.11
2018	221.62	295.49	295.49	295.49	369.37	\$1,477.46



PRINCIPLES OF CLAIMS-MADE POLICIES

■ Principle 3 : (conti.)

□ Occurrence Loss Costs

Report Year	Report Lag					Occurrence Loss Costs
	0	1	2	3	4	
2010	150.00	200.00	200.00	200.00	250.00	
2011	157.50	210.00	210.00	210.00	262.50	
2012	165.38	220.50	220.50	220.50	275.63	
2013	173.64	231.53	231.53	231.53	289.41	
2014	182.33	243.10	243.10	243.10	303.88	
2015	191.44	255.26	255.26	255.26	319.07	\$1,115.91
2016	201.02	268.02	268.02	268.02	335.03	\$1,171.70
2017	211.07	281.42	281.42	281.42	351.78	\$1,230.30
2018	221.62	295.49	295.49	295.49	369.37	\$1,291.80
						\$1,356.40

25



PRINCIPLES OF CLAIMS-MADE POLICIES

■ Principle 3 : (conti.)

- There is no impact on the loss cost estimates for the claims-made policies
- The estimates for the occurrence policies have changed from the original table
 - The Accident Year 2011 loss cost estimate for the occurrence policies has changed by 1% (= $(\$1,171.70 / \$1,160.39) - 1.0$)

26



PRINCIPLES OF CLAIMS-MADE POLICIES

- **Principle 4 :**

Claims-made policies incur no liability for IBNR, so the risk of reserve inadequacy is greatly reduced



PRINCIPLES OF CLAIMS-MADE POLICIES

- **Principle 4 : (conti.)**

- For occurrence policies

- need to estimate claims that are incurred but not reported (pure IBNR) and claims that are incurred but not enough reported (IBNER)

- For claims-made policies

- no pure IBNR component; only need to determine an IBNER reserve
- the risk of reserve inadequacy is greatly reduce



PRINCIPLES OF CLAIMS-MADE POLICIES

- **Principle 5 :**

The investment income earned from claims-made policies is substantially less than under occurrence policies



PRINCIPLES OF CLAIMS-MADE POLICIES

- **Principle 5 : (conti.)**

- Claims-made policy shortens the period of time between collection of premium and payment of claim; consequently, funds invested for a shorter time horizon result in less investment income
- When determining the target underwriting profit provision for a claims-made policy, the pricing actuary should take into consideration both the reduced investment income as well as the reduced pricing risk



COORDINATING POLICIES

- Since occurrence and claims-made policies have different coverage triggers, insureds converting from one policy type to the other should be cognizant of coverage **overlaps** or **gaps**



COORDINATING POLICIES

- Example
 - As shown in the following diagram, there is overlapping coverage between the occurrence policy and the claims-made policy for an insured switches from **occ** to **c-m** starting in 2011

Report Year	Report Lag				
	0	1	2	3	4
2010	L(2010,0)	L(2010,1)	L(2010,2)	L(2010,3)	L(2010,4)
2011	L(2011,0)	L(2011,1)	L(2011,2)	L(2011,3)	L(2011,4)
2012	L(2012,0)	L(2012,1)	L(2012,2)	L(2012,3)	L(2012,4)
2013	L(2013,0)	L(2013,1)	L(2013,2)	L(2013,3)	L(2013,4)
2014	L(2014,0)	L(2014,1)	L(2014,2)	L(2014,3)	L(2014,4)
2015	L(2015,0)	L(2015,1)	L(2015,2)	L(2015,3)	L(2015,4)



COORDINATING POLICIES

- Retroactive Date
 - The claims-made coverage only covers claims that occur on or after the retroactive date
 - To provide complete coverage without overlap, the retroactive date should be coordinated with the expiration of the last occurrence policy
 - The insured can purchase :
 - a first-year claims-made policy in 2011 with a retroactive date of January 1, 2011, and
 - a second-year claims-made policy with a retroactive date of January 1, 2011,
 - ,..., and a mature claims-made policy in 2015



COORDINATING POLICIES

- Retroactive Date (conti.)
 - The first-year claims-made policy will cover L(2011,0)
The second-year claims-made policy will cover L(2012,0) and L(2012,1)
The mature claims-made policy will cover L(2015,0) to L(2015,5)

Report Year	Report Lag				
	0	1	2	3	4
2010	L(2010,0)	L(2010,1)	L(2010,2)	L(2010,3)	L(2010,4)
2011	L(2011,0)	L(2011,1)	L(2011,2)	L(2011,3)	L(2011,4)
2012	L(2012,0)	L(2012,1)	L(2012,2)	L(2012,3)	L(2012,4)
2013	L(2013,0)	L(2013,1)	L(2013,2)	L(2013,3)	L(2013,4)
2014	L(2014,0)	L(2014,1)	L(2014,2)	L(2014,3)	L(2014,4)
2015	L(2015,0)	L(2015,1)	L(2015,2)	L(2015,3)	L(2015,4)



COORDINATING POLICIES

- The Step Factor
 - is used to recognize the growth in exposure for each successive claims-made policy during the transition
 - is a percentage of the mature claims-made rate
 - The cumulative values of the loss estimates for L(2015,0), L(2015,1), L(2015,2), L(2015,3) and L(2015,4) expressed as a ratio to the total losses for Report Year 2015 can be used to determine the step factors

Claims-Made Year	Step Factor
First	40%
Second	70%
Third	85%
Fourth	95%
Fifth or More	100%



COORDINATING POLICIES

- Example
 - Switching from a claims-made policy to an occurrence policy in 2011
 - This situation may creates a coverage gap (i.e., no coverage for claims that occurred before 2011 but were not reported)

Report Year	Report Lag				
	0	1	2	3	4
2010	L(2010,0)	L(2010,1)	L(2010,2)	L(2010,3)	L(2010,4)
2011	L(2011,0)	L(2011,1)	L(2011,2)	L(2011,3)	L(2011,4)
2012	L(2012,0)	L(2012,1)	L(2012,2)	L(2012,3)	L(2012,4)
2013	L(2013,0)	L(2013,1)	L(2013,2)	L(2013,3)	L(2013,4)
2014	L(2014,0)	L(2014,1)	L(2014,2)	L(2014,3)	L(2014,4)
2015	L(2015,0)	L(2015,1)	L(2015,2)	L(2015,3)	L(2015,4)



COORDINATING POLICIES

- Example (conti.)
 - Companies offer an extended reporting endorsement (or tail coverage) that covers claims that occurred but were not reported before the expiration of the last claims-made policy

Report Year	Report Lag				
	0	1	2	3	4
2010	L(2010,0)	L(2010,1)	L(2010,2)	L(2010,3)	L(2010,4)
2011	L(2011,0)	L(2011,1)	L(2011,2)	L(2011,3)	L(2011,4)
2012	L(2012,0)	L(2012,1)	L(2012,2)	L(2012,3)	L(2012,4)
2013	L(2013,0)	L(2013,1)	L(2013,2)	L(2013,3)	L(2013,4)
2014	L(2014,0)	L(2014,1)	L(2014,2)	L(2014,3)	L(2014,4)
2015	L(2015,0)	L(2015,1)	L(2015,2)	L(2015,3)	L(2015,4)

37



COORDINATING POLICIES

- The gap in coverage in the case of retirement
 - The protection is given by a tail policy that covers losses occurring during the period for which claims-made coverage was in force and that are reported after the insured's last claims-made policy expires

Report Year	Report Lag				
	0	1	2	3	4
2010	L(2010,0)	L(2010,1)	L(2010,2)	L(2010,3)	L(2010,4)
2011	L(2011,0)	L(2011,1)	L(2011,2)	L(2011,3)	L(2011,4)
2012	L(2012,0)	L(2012,1)	L(2012,2)	L(2012,3)	L(2012,4)
2013	L(2013,0)	L(2013,1)	L(2013,2)	L(2013,3)	L(2013,4)
2014	L(2014,0)	L(2014,1)	L(2014,2)	L(2014,3)	L(2014,4)
2015	L(2015,0)	L(2015,1)	L(2015,2)	L(2015,3)	L(2015,4)

38



KEY CONCEPTS

- Rationale for claims-made coverage
- Aggregating losses by report year and report lag
- Coverage triggers for claims-made coverage
- Five principles of claims-made policies
- Coordinating coverage
 - Retroactive date
 - First- and second-year claims-made policies
 - Mature claims-made policies
 - Extended reporting endorsement or tail coverage

EXAM 5, SPRING 2012

8. (2 points)

A physician maintained medical malpractice coverage with occurrence policies through 2011. Effective January 1, 2012, the physician switched to claims-made coverage. The physician will retire on December 31, 2014. The last claims-made policy to be issued prior to the physician's retirement date will be effective from January 1, 2014 to December 31, 2014.

The following table contains anticipated loss costs used to evaluate pricing for the physician's policy. All claims are reported within 3 years.

Report Year	Report Year Lag			
	0	1	2	3
2011	\$350	\$300	\$250	\$100
2012	\$368	\$315	\$263	\$105
2013	\$386	\$331	\$276	\$110
2014	\$405	\$347	\$290	\$116
2015	\$426	\$365	\$304	\$122
2016	\$447	\$383	\$319	\$128
2017	\$469	\$402	\$335	\$134

a. (0.5 point)

Briefly describe two advantages that claims-made coverage has over occurrence coverage for a medical malpractice insurer.

b. (1 point)

Calculate the loss costs associated with a 2011 occurrence policy and the loss costs associated with a mature 2012 claims-made policy. Briefly describe the overlap in loss costs between the two and the mechanism used to prevent it.

c. (0.5 point)

Identify the loss costs in the table above for which the physician would still have exposure at the time of retirement, and the coverage that the physician would need to purchase to transfer that exposure to the insurer.

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Exam 5 Question 8 (Exam 5A Question 8)

a. 1. Med Mal has a very long tail, so for occurrence policies it takes a very long time to develop. For claims made policies losses are known at the end of the year.

No Pure IBNR component needed.

2. Since CM policies have a shorter time frame, they would be less subject to changes in trend or inflation than occurrence policies.

b. 2011 Occurrence = $350 + 315 + 276 + 116 = 1057$

2012 Mature CM = $368 + 315 + 263 + 105 = 1051$

Overlap would be for RY2012 Lag 1- the 315 would be covered by both. To prevent this, CM policies have retro dates which signal the beginning of coverage (losses that occurred before retro date would not be covered, only losses that occurred after). So after the occurrence policy in 2011, the CM policy in 2012 should have a retro date of 1/1/2012 + be a first year CM policy.

c. Loss costs after would be $365 + 304 + 122 + 319 + 128 + 134 = 1372$

There would be losses that occurred while the CM policies were still in place but were reported after the physician retired. Physician would need to purchase a tail coverage to cover these losses.

a. Many candidates received full credit on this part. Some candidates did not provide enough detail to receive credit, using statements like "less pricing risk" and "less reserving risk". Other candidates provided slightly different variations on the same item.

b. Most candidates did identify the correct loss costs. Full credit was given for identifying the overlap graphically. Some candidates lost credit for not addressing the specific overlap for this question.

c. Many candidates were able to identify the loss costs associated with the tail exposure. Some responses were not able to identify tail coverage, instead listing some combination of claims made or occurrence policies that did not match the exposure.

EXAM 5, SPRING 2011

22. (2.25 points)

Given the following claim detail (000s):

<u>Claim</u>	<u>Accident Date</u>	<u>Paid During 2008</u>	<u>Case Reserve at Year End 2008</u>	<u>Paid During 2009</u>	<u>Case Reserve at Year End 2009</u>	<u>Paid During 2010</u>	<u>Case Reserve at Year End 2010</u>
1	March 3, 2008	\$225	\$190	\$250	\$0	\$0	\$0
2	July 18, 2008	\$150	\$500	\$0	\$500	\$230	\$270
3	December 1, 2008			\$105	\$75	\$75	\$25
4	March 1, 2009			\$200	\$200	\$150	\$100
5	October 3, 2009			\$320	\$280	\$200	\$0
6	November 3, 2009			\$0	\$100	\$50	\$100
7	April 12, 2010					\$45	\$55
8	June 28, 2010					\$500	\$500

a. (1.25 points)

Construct the cumulative accident year reported loss development triangle as of December 31, 2010.

b. (0.5 point)

Describe a situation in which it is preferable to use accident year data for estimating unpaid claims rather than report year data.

c. (0.5 point)

Describe a situation in which it is preferable to use report year data for estimating unpaid claims rather than policy year data.

CONTINUE ON NEXT PAGE

Question 22

a)

Incremental Paid Loss (as of months)

AY	12	24	36
08	375	355	305
09	520	400	
10	545		

Case reserve (as of mths)

AY	12	24	36
08	690	575	295
09	580	200	
10	555		

Cumulative Rpt Loss (as of mths) = cum. Paid loss + Case reserve

AY	12	24	36
08	1065	1305	1330 = 375+355+305+295
09	1100	1120	
10	1100		

b) Sample1

When you want to estimate IBNR – this is impossible (or at least very difficult) with report year data, since unpaid claims for a report year are IBNER, not pure IBNR.

Sample 2

AY data is widely used to estimate unpaid claims so there are many industry benchmarks and data based on AY aggregation. When the actuary wants to use such benchmarks or industry data, then it is preferable to use internal AY data as well.

c) Sample1

When you are modeling claims made policies, it makes sense to use report year, since CM policies operate on report years and have no IBNR at year end.

Sample 2

When there is a severe change in legal or social climate that the average severity tracks more with the report date rather than the occurrence/accident date of the policy.

EXAM 5, SPRING 2010

22. (2 points)

a. (0.5 point)

Explain the major difference between claims-made and occurrence policies.

b. (0.5 point)

Explain how claims-made coverage reduces pricing risk.

c. (0.5 point)

Explain how claims-made coverage reduces reserving risk.

d. (0.5 point)

Explain the purpose of an extended reporting endorsement (or tail policy).

CONTINUED ON NEXT PAGE

Question 22

a. Coverage Trigger

Claims made coverage is triggered when claim is first reported to the insurer, (given that it occurred on or after the retroactive date and is reported in policy period)

Occurrence coverage is triggered when the loss occurs during the policy period.

b. Claims made policies have a much shorter tail than occurrence policies, so they are not affected as severely by inflation, trend, etc. This makes pricing the future easier.

c. There is no IBNR (incurred but not reported) on a claims-made policy since all claims are reported at the end of policy period. So, it makes it easier to set reserves.

d. To provide coverage for claims that may be reported after the claims-made policy expires.

For example, a doctor may retire, so no more new claims occur, but he/she needs coverage for claims that might be reported after he/she is done practicing medicine.

EXAM 5, SPRING 2009

28. (2 points)

An insured has purchased the following policies:

Effective Date	Policy Term	Policy Type	Retroactive Date
January 1, 2004	1 Year	Occurrence	N/A
January 1, 2005	1 Year	Occurrence	N/A
January 1, 2006	1 Year	First-Year Claims Made	January 1, 2006
January 1, 2007	1 Year	Second-Year Claims Made	January 1, 2006
January 1, 2008	1 Year	Third-Year Claims Made	January 1, 2006

A tail policy is also purchased on January 1, 2009 to cover any losses that occurred while the claims-made policies were in effect but had not been reported as of December 31, 2008.

Draw and label a diagram that shows what losses each policy covers, based on when the losses occurred and when they were reported, assuming all claims are reported within 3 years of occurrence.

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Question#: 28

Report Year								
LAG	2004	2005	2006	2007	2008	2009	2010	2011
0	A	B	C	D	E			
1		A	B	D	E	F		
2			A	B	E	F	F	
3				A	B	F	F	F

A: Occurrence Policy Effective 1/1/2004

B: Occurrence Policy Effective 1/1/2005

C: 1st year CM Policy 1/1/2006

D: 2nd year CM Policy Effective 1/1/2007

E: 3rd year CM Policy Effective 1/1/2008

F: Tail Policy Effective 1/1/2009

EXAM 5, SPRING 2008

20. (2.0 points)

Using the techniques contained in Marker and Mohl, "Rating Claims-Made Insurance Policies", draw and label a diagram representing the following five different types of policies written in 2000 through 2004. Assume all claims are reported within four years.

- a. Occurrence policy written in year 2000
- b. 1st year claims-made policy written in 2001
- c. 2nd year claims-made policy written in 2002
- d. 3rd year claims-made policy written in 2003
- e. Tail policy written in 2004

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Exam 5, Question 20

Lag	Report Year						
	2000	2001	2002	2003	2004	2005	2006
0	A	B	C	D			
1		A	C	D	E		
2			A	D	E	E	
3				A	E	E	E...
4					A	E	E...
5						A	E...
6							A...

- a. Occurrence policy in \$2000 = All A's
- b. 1st year claims made = B
- c. 2nd year claims made = All C's
- d. 3rd year claims made = All D's
- e. 2004 tail policy = All E's